BVRIT HYDERABAD
College of Engineering for Women

Approved by AICTE and Affiliated to JNTUH, Hyderabad
Accredited by NBA & NAAC (A Grade)
Rajiv Gandhi Nagar, Bachupally, HYDERABAD – 500090
Telangana, India

COURSE CONTENT	
Department	Basic Sciences and Humanities
Year/Semester	I B.Tech. / I Semester
Subject	Matrices & Calculus
Regulation	R22

VISHNU
UNIVERSAL LEARNING

VISION
To emerge as the best among the institutes of technology and research in the country dedicated to the cause of promoting quality technical education.

MISSION

At BVRITH, we strive to

- Achieve academic excellence through innovative learning practices.
- Enhance intellectual ability and technical competency for a successful career.
- Encourage research and innovation.
- Nurture students towards holistic development with emphasis on leadership skills, life skills and human values.

MATRICES AND CALCULUS

B.Tech. I Year I Sem. **L T P C** 3 1 0 4

Pre-requisites: Mathematical Knowledge at pre-university level

Course Objectives: To learn

- Types of matrices and their properties.
- Concept of a rank of the matrix and applying this concept to know the consistency and solvingthe system of linear equations.
- Concept of eigenvalues and eigenvectors and to reduce the quadratic form to canonical form
- Geometrical approach to the mean value theorems and their application to the mathematicalproblems
- Evaluation of surface areas and volumes of revolutions of curves.
- Evaluation of improper integrals using Beta and Gamma functions.
- Partial differentiation, concept of total derivative
- Finding maxima and minima of function of two and three variables.
- Evaluation of multiple integrals and their applications

Course outcomes: After learning the contents of this paper the student must be able to

- Write the matrix representation of a set of linear equations and to analyze the solution of thesystem of equations
- Find the Eigenvalues and Eigen vectors
- Reduce the quadratic form to canonical form using orthogonal transformations.
- Solve the applications on the mean value theorems.
- Evaluate the improper integrals using Beta and Gamma functions
- Find the extreme values of functions of two variables with/ without constraints.
- Evaluate the multiple integrals and apply the concept to find areas, volumes

UNIT-I: Matrices **10 L**

Rank of a matrix by Echelon form and Normal form, Inverse of Non-singular matrices by Gauss- Jordan method, System of linear equations: Solving system of Homogeneous and Non-Homogeneous equations by Gauss elimination method, Gauss Seidel Iteration Method.

UNIT-II: Eigen values and Eigen vectors **10 L**

Linear Transformation and Orthogonal Transformation: Eigenvalues, Eigenvectors and theirproperties, Diagonalization of a matrix, Cayley-Hamilton Theorem (without proof), finding inverse and power of a matrix by Cayley-Hamilton Theorem, Quadratic forms and

Nature of the Quadratic Forms, Reduction of Quadratic form to canonical forms by Orthogonal Transformation.

UNIT-III: Calculus **10 L**

Mean value theorems: Rolle's theorem, Lagrange's Mean value theorem with their Geometrical Interpretation and applications, Cauchy's Mean value Theorem, Taylor's Series.

Applications of definite integrals to evaluate surface areas and volumes of revolutions of curves (Only in Cartesian coordinates), Definition of Improper Integral: Beta and Gamma functions and their applications.

UNIT-IV: Multivariable Calculus (Partial Differentiation and applications) **10 L**

Definitions of Limit and continuity.

Partial Differentiation: Euler's Theorem, Total derivative, Jacobian, Functional dependence & independence. Applications: Maxima and minima of functions of two variables and three variables using method of Lagrange multipliers.

UNIT-V: Multivariable Calculus (Integration) **8 L**

Evaluation of Double Integrals (Cartesian and polar coordinates), change of order of integration (only Cartesian form), Evaluation of Triple Integrals: Change of variables (Cartesian to polar) for double and (Cartesian to Spherical and Cylindrical polar coordinates) for triple integrals.

Applications: Areas (by double integrals) and volumes (by double integrals and triple integrals).

TEXT BOOKS:

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010.
2. R.K. Jain and S.R.K. Iyengar, Advanced Engineering Mathematics, Narosa Publications,5th Editon, 2016.

REFERENCE BOOKS:

1. Erwin kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
2. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9thEdition,Pearson, Reprint,2002.
3. N.P. Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications,Reprint, 2008.
4. H. K. Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S Chand and CompanyLimited, New Delhi

BVRIT HYDERABAD
College of Engineering for Women
Bachupally, Hyderabad – 500090
Department of Basic Science and Humanites
B.Tech I Year Sec-A/B/C **LESSON PLAN (R22)**

Course Code: **Course Title: M&C**
Class: I CSE/ECE/EEE/IT/CSE- AIML **Academic Year: 2022-23**

UNIT – I: Matrices:

Types of Matrices, Real matrices: Symmetric, skew–symmetric and orthogonal matrices. Complex matrices: Hermitian, Skew-Hermitian, Unitary and Idempotent matrices. Elementary matrix, Finding rank of a matrix by reducing to Echelon and Normal forms. Finding the inverse of a non-singular square matrix using row/column transformations (Gauss- Jordan method). System of linear equations homogeneous and non-homogeneous equations. Gauss elimination Method, Gauss –Seidel iteration Method

Session No.	Date	Topic Proposed to be Covered	Text /Reference Book	Chapter No. & Page No.	Web Resources	COs Achieved
1		Introduction about Matrices, Applications	T1 O1	2 & 26,27 1&2	1.nptel.ac.in/courses/122104018/ 2.nptel.ac.in/courses/122107036/15 3.nptel.ac.in/courses/122107036/16	The significance of Matrices and real time applications known
2		**Real matrices:** Symmetric, skew – symmetric and, orthogonal matrices.	T1 O1	2&26-35, 1&7-8		
3		**Complex matrices:** Hermitian, Skew-Hermitian,Unitary and Idempotent	T1 O1	2&67-71, 1& 6		
4		Elementary row and column transformations- Elementary matrix	T1 O1	2&36-40, 1& 25,49		
5		Finding rank of a matrix by reducing to Echelon form matrices. reducing to Normal form	T1 O1	2&35-42, 1& 38-49		
6		System of Linear Equations	T1,O1	2&46-51 1&67-112		
7		Consistency of system of linear equations (homogeneous and non- homogeneous) using the rank of a matrix	T1,O1	2&46-51 1&67-112		
8		Gauss elimination method	T1,O1	350-354 327-330		

9		Inverse of a non-singular square matrix using row/ column transformations	T1,O 1	2&35-42 1& 59-66		
10		Gauss Seidel Method	T1			
11		Review of previous years question papers	T1			
12		Tutorial	T1,T 2			
1		Activity- Quiz				

UNIT-II : Eigen values and Eigen vectors :Linear Transformation – Orthogonal Transformation: Eigen values and eigen vectors of a matrix. Properties of eigen values and eigen vectors of real and complex matrices. Diagonalization of matrix **,**Cayley-Hamilton Theorem (without proof) – Finding inverse of a matrix and powers of a matrix by Cayley-Hamilton theorem.. Quadratic forms and nature of the Quadratic forms; Reduction of a quadratic form to canonical forms by Orthogonal Transformation

13		Eigen values and eigen Vectors of matrix –properties	T2 O1	2& 54-58 2&149-183	1. https://onlinecourses.nptel.ac.in/noc18_ma14 2. nptel.ac.in/courses/122107036/27 3. nptel.ac.in/courses/108108079/pdf/Unit%203/Unit_3.1.pdf	Real time applications of Matrices using eigen values and eigen vectors identified.
14		Inverse of a matrix by using Cayley Hamilton theorem	T2 T1	58-60 58-61		
15		Powers of a matrix using Cayley Hamilton theorem(without Proof)	T2 T1	62-64 58-64		
16		Diagonalization of matrix	T2 O1 T1	2& 61-64 61-64		
17		Quadratic forms	T1	2&64-67		
18		Nature of the quadratic forms	O1	4&253-264		
19		Reduction of Quadratic form to canonical forms	T1 O1	2&64-67		
20		Orthogonal transformation	O1	4&265-276		
21		Problems related to transformation	T1 O1	2&64-67		
22		Linear transformation	T1 O1	2&64-67		
23		Reduction of Quadratic form to canonical forms by Orthogonal transformation	T1 O1	2&64-67 4&276-298		
24		**Review of previous years question papers**				

25		**Tutorial**	**T1 ,T2**			
2		Activity- Group Discussion				

UNIT – III: Calculus

Mean value theorems: Rolles theorem ,Lagrange's Mean value theorem with their Geometrical Interpretation and Applications , Cauchy's Mean value Theorem ,Taylors Series. Applications of definite Integrals to evaluate Surface areas and volumes of revolutions of curves (only in cartesian coordinates), Definition of Improper Integral : Beta and Gamma functions and their applications**.**

26		Rolles theorem with their Geometrical Interpretation and Applications	T1	365-385	1.nptel.ac.in/courses/111106053/46	Analyze the nature of Mean value theorems and also Special functions
27		Lagrange's Mean value theorem with their Geometrical Interpretation and Applications	T1	368		
28		Cauchy's Mean value Theorem with their Geometrical Interpretation and Applications	T1	376		
29		Taylors Series and Applications	T1	378		
30		Applications of definite Integrals to evaluate Surface areas	T1	380	2.nptel.ac.in/courses/111105069/19	
31		volumes of revolutions of curves (only in cartesian coordinates)	O1	382	3. nptel.ac.in/courses/122104017/	
32		Definition of Improper Integral	T1	383		
33		Beta functions and their applications**.**	T1	383		
34		Gamma functions and their applications**.**	T1	385		
35		**Review of previous years question papers**				
		Tutorial	T1,T2			
3		Activity- Group Discussion				

UNIT – IV: Multi Variable Calculus (Partial Differentiation and Applications) Definitions of Limit and Continuity. Partial differentiation: Euler's theorem , total derivative , Jacobian , Functional dependence and independence Applications: Maxima and Minima of functions of two variables and three variables using method of Lagrange multipliers

36		Definition of a Limit and continuity,PDE	T1		.https://onlinecourses.nptel.ac.in/noc18_ma05/course 2.https://onlinecourses.nptel.ac.in/noc18_ma05/announcements 3.nptel.ac.in/Clarifydoubts.php?subjectId=111107108	Finding the extreme values of functions of two variables with /without constraints
37		Partial differentiation	T1			
38		Eulers theorem	T1			
39		Total derivative	T1 T1			
40		Jacobian	T1	142-151		
41		Functional dependency	T1			
42		Functional independency	T1	274 -290		
43		Maxima and minima of functions of two variables using Method of Lagrange's multipliers	T1	274 -302		
44		Maxima and minima of functions of three variables using Method of Lagrange's multipliers	T1			
45		**Review of previous years question papers**				
46		**Tutorial**	T1,T 2	274-309		
4		Activity – Chart Preparation				

UNIT – V : Multivariable Calculus (Integration) 8 L

Evaluation of Double Integrals (Cartesian and polar coordinates), change of order of integration (only Cartesian form), Evaluation of Triple Integrals: Change of variables (Cartesian to polar) for double and(Cartesian to Spherical and Cylindrical polar coordinates) for triple integrals.
Applications: Areas (by double integrals) and volumes (by double integrals and triple integrals).

47		Introduction to Multiple integrals and its applications	T1		1.nptel.ac.in/courses/122104017/28	. Apply the concept of Multiple Integral to find areas, volumes,
48		Multiple integrals-Double integrals	T1			
49		Triple integrals	T1			
50		Evaluation of double integrals (Cartesian and Polar coordinates)	T1 T1			
51		Double integrals :Change of order of integration(Cartesian form)	T1	198-233		

52		Evaluation of triple integrals; Change of Variables(Cartesian to polar) for double	T1		2. nptel.ac.in/courses/111107108/28	for cubes, sphere and rectangular parallelepiped.
53		Evaluation of triple integrals; Change of Variables(Cartesian to Spherical and Cylindrical polar coordinates) for triple integrals	T1			
54		Finding the area of region using double integration	T1			
55		Finding the volume of a region using triple integration	T1			
56		Review of previous years question papers				
57		**Review of previous years question papers**	**T1 ,T2**			
58		**Tutorial**		200-233		
59		**Activity- Mind Map**				

TEXT BOOKS:

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010.
2. R.K. Jain and S.R.K. Iyengar, Advanced Engineering Mathematics, Narosa Publications, 5th Editon, 2016.

REFERENCE BOOKS:

R1. Advanced Engineering Mathematics by R.K. Jain & S.R.K. Iyengar, 3rd edition, Narosa Publishing House, Delhi.

R2. Engineering Mathematics – I by T.K. V. Iyengar, B. Krishna Gandhi & Others, S. Chand.

R3. Engineering Mathematics – I by D. S. Chandrasekhar, Prison Books Pvt. Ltd.

R4. Engineering Mathematics – I by G. Shanker Rao & Others I.K. International Publications.

R5. Advanced Engineering Mathematics with MATLAB, Dean G. Duffy, 3rd Edi, CRC Press

R6.N.P.Bali and Manish Goyal,A text book of Engineering Mathematics , Lakshmi Publications,Reprint, 2008.

R7.Higher Engineering Mathematics B.V.Ramana, Tata McGraw Hill New Delhi,11 th Reprint, 2010.

R8. Advanced Engineering Mathematics, Michael Greenberg, Second Edition. Pearson Education.

R9. N.P.Bali and Manish Goyal , A text book of Engineering Mathematics , Laxmi Publications,Reprint, 2008.

R10. Erwin kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
R.11 .H. K. Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S Chand and CompanyLimited,

OTHER REFERENCE BOOKS:

O1. Engineering Mathematics-1 by T.K.V. Iyengar, B.Krishna Gandhi & Others, S.Chand , Vol-1
O2. Engineering Mathematics – II by T.K. V. Iyengar, B. Krishna Gandhi & Others, S.Chand,Vol-2

Signature of Faculty **HOD**

Course Outcomes

Matrices & Calculus	Course Outcomes	Bloom's Taxonomy
C111.1	Solve the system of linear equations using appropriate methods	Apply
C111.2	Analyze the nature of quadratic form using eigen values and eigen vectors	Analyze
C111.3	Derive infinite series expansions of differentiable functions using generalized mean value theorems	Apply
C111.4	Evaluate improper integrals using Beta and Gamma functions	Apply
C111.5	Optimize a given function with respect to given constrains	Analyze
C111.6	Estimate area or volumes of few geometries using multiple integration	Apply

SOME USEFUL FORMULAE FROM INTERMEDIATE

TRIGNOMETRIC FORMULAE:

θ	0°	30°	45°	60°	90°
sin θ	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = co\sec^2 x$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^3 x = \frac{1}{4}[3\sin x - \sin 3x] \quad \cos^3 x = \frac{1}{4}[3\cos x + \cos 3x]$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B) \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\cosh ax = \frac{e^{ax} + e^{-ax}}{2}$$

$$\sinh ax = \frac{e^{ax} - e^{-ax}}{2}$$

DIFFERENTIATION FORMULAE:

$$\frac{d}{dx}(K)=0$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}(a^x)=a^x\log a$$

$$\frac{d}{dx}(e^x)=e^x$$

$$\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}$$

$$\frac{d}{dx}(\log x)=\frac{1}{x}$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\cot x)=-\cos ec^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\ \tan x$$

$$\frac{d}{dx}(\cos ecx)=-\cos ecx\ \cot x$$

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos ec^{-1}x)=-\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \sec h^2 x$

$\frac{d}{dx}(\coth x) = -\cos ech^2 x$

$\frac{d}{dx}(Ku) = K\frac{d}{dx}(u)$

$\frac{d}{dx}(u+v) = \frac{d}{dx}(u) + \frac{d}{dx}(v)$

$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$

$\frac{d}{dx} f[g(x)] = f'[g(x)] \times g'(x)$

Partial Differentiation

If $U(x, y)$ is a function of two variables then

(i) partial differentiation of $U(x, y)$ wrto x partially means differentiation of $U(x, y)$ considering y as constant. It is denoted $\frac{\partial U}{\partial x}$.

(ii) partial differentiation of $U(x, y)$ wrto y partially means differentiation of $U(x, y)$ considering x as constant. It is denoted $\frac{\partial U}{\partial y}$.

INTEGRATION FORMULAE:

$\int k\ dx = k\ x + c$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

$\int \frac{1}{x} dx = \log|x| + c$

$\int \log x dx = x \log|x| - x + c$

$$\int a^x dx = \frac{a^x}{\log a} + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cos} ec^2 x \text{ dx} = -\text{cotx} + \text{c}$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\int \cos ecx \cot x dx = -\cos ecx + c$$

$$\int \tan x dx = -\log|\cos x| + c \text{ or}$$

$$\log|\sec x| + c$$

$$\int \cot x dx = \log|\sin x| + c$$

$$\int \sec x dx = \log|\sec x + \tan x| + c \text{ or } \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

$$\int \cos ecx dx = \log|\cos ecx - \cot x| + c \ or \ \log\left|\tan\frac{x}{2}\right| + c$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int \tanh x dx = \log\cosh x + c$$

$$\int \coth x dx = \log\sinh x + c$$

$$\int \operatorname{sech}^2 x dx = \tanh x + c$$

$$\int \cos ec\text{h}^2 x dx = -\coth x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + \text{c} \ or \ -\cos^{-1} x + \text{c}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \ \ or -\cot^{-1} x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1} x + c \quad or \quad -\operatorname{cosec}^{-1} x + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c \ or \ \ -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c \ or \ -\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c \ or \ -\frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)$$

INTEGRATION BY PARTS:

Integration by parts is used in integrating product of functions of the type $f(x).g(x)$ as follows:

$$\int\left(I^{st}\ function\times II^{nd}\ function\right)dx=I^{st}\ function\int\left(II^{nd}\ function\right)dx$$
$$-\int\left(\frac{d}{dx}\left(I^{st}\ function\right)\times\int\left(II^{nd}\ function\right)dx\right)dx$$

Where the I[st] and II[nd] functions are decided in the order of ***ILATE***;

I: Inverse trigonometric function

L: Logarithmic function

T: Trigonometric functions

A: Algebraic functions

E: Exponential Functions

I $\int\left[f_1(x)\pm f_2(x)\right]dx=\int f_1(x)\,dx\pm\int f_2(x)\,dx$

II $\int k.f(x)\,dx=k\int f(x)\,dx$

III $\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+c, n\neq -1$

$$\int\frac{1}{ax+b}dx=\frac{\log|ax+b|}{a}+c$$

$$\int e^{ax+b}dx=\frac{e^{ax+b}}{a}+c$$

$$\int\sin(ax+b)\,dx=-\frac{\cos(ax+b)}{a}+c$$ **etc**

IV $\int\left[f(x)\right]^n.f'(x)\,dx=\frac{\left[f(x)\right]^{n+1}}{n+1}+c$

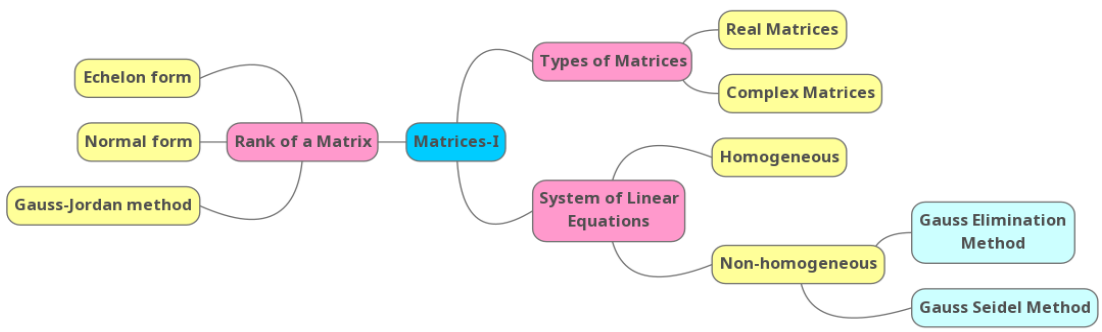
V $\int\left(\frac{f'(x)}{f(x)}\right)dx=\log|f(x)|+c$

VI $\int\left(\frac{f'(x)}{\sqrt{f(x)}}\right)dx=2\sqrt{f(x)}+c$

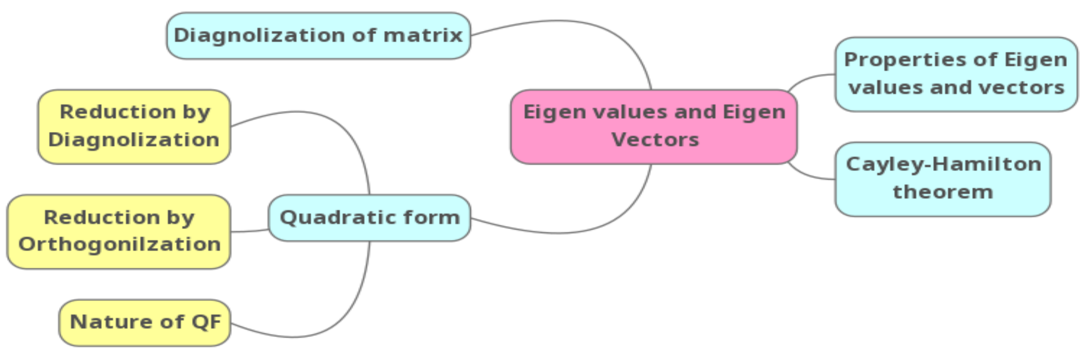
VII $\int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+c$ **VIII** $\int e^{f(x)}f'(x)dx=e^{f(x)}+c$

Mind Maps

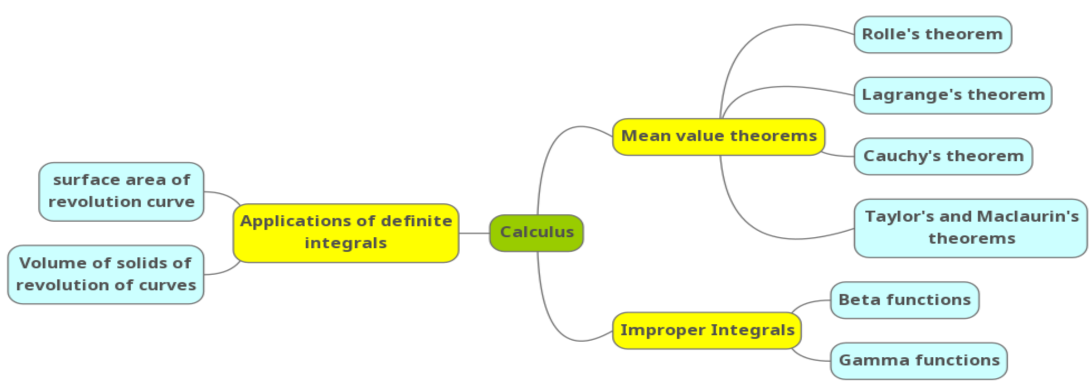
Unit I



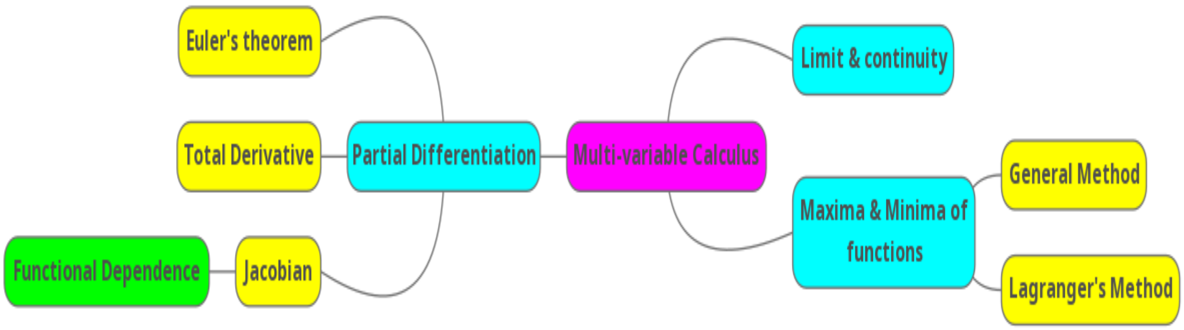
Unit II



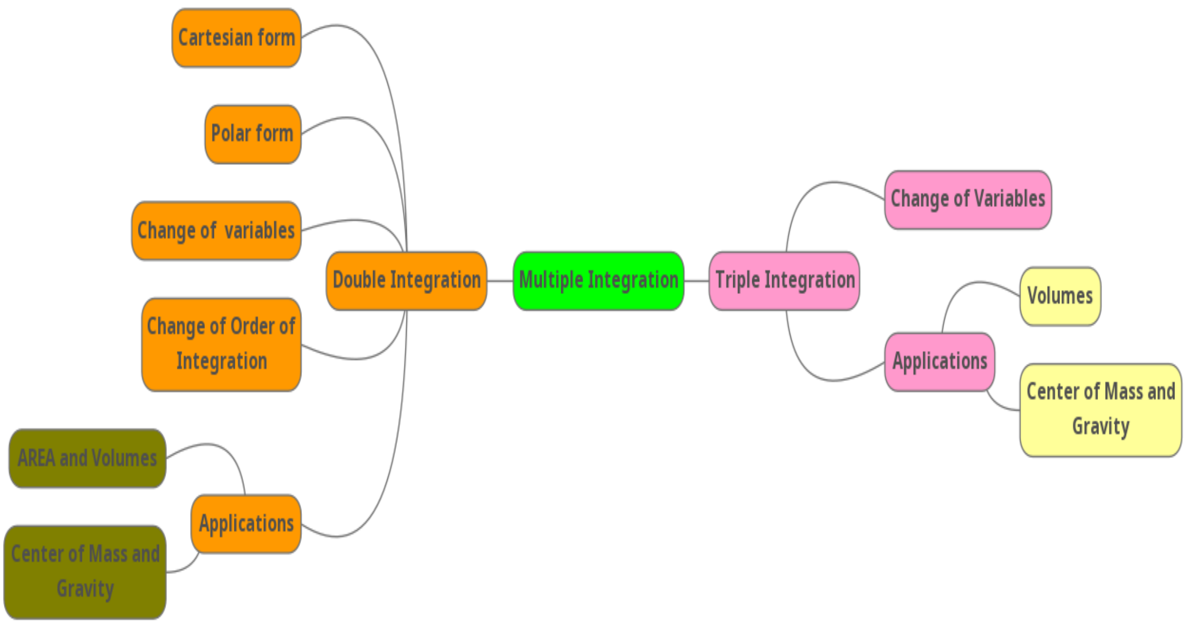
Unit III



Unit IV



Unit V



Key Points
UNIT I Theory of Matrices

Identity matrix :

If I= $[a_{ij}]_{nxn}$ such that $a_{ij} = 1$ for i = j and $a_{ij} = 0$ for $i \neq j$, then I is called a identity matrix.

Zero matrix :

If A = $[a_{ij}]_{mxn}$ that $a_{ij} = 0 \quad \forall$ i, j then A is called a zero matrix (or) null matrix.

Diagonal elements in a matrix

A= $[a_{ij}]_{mxn}$, the elements a_{ij} of A for which i = j i.e. (a_{11}, a_{22}….a_{nn}) are called the diagonal elements of A.

- The line along which the diagonal elements lie is called the principle diagonal of A

Diagonal matrix :

A square matrix all of whose elements except those in leading diagonal are zero is called diagonal matrix.

- If d_1, d_2….. d_n are diagonal elements of a diagonal matrix, A, then A is written as A = diag (d_1,d_2….d_n)

The transpose of a matrix:

The matrix obtained from any given matrix A, by interchanging its rows and columns is called the transpose of A. It is denoted by A′ (or) A^T.

UpperTriangular matrix :

A square matrix all of whose elements below the leading diagonal are zero is called an Upper triangular matrix.

Lower triangular matrix ;

A square matrix all of whose elements above the leading diagonal are zero is called a lower triangular matrix

Real and complex matrices

Symmetric matrix : Thus A is a symmetric matrix if $A^T = A$

Skew – Symmetric : A square matrix A is said to be skew – symmetric if $A^T = -A$

- Every diagonal element of a skew – symmetric matrix is necessarily zero.

Trace of a square matrix : The sum of diagonal elements of a matrix is called trace of the matrix.

Idempotent matrix : If A is a square matrix such that $A^2 = A$ then 'A' is called idempotent matrix

Nilpotent Matrix : If A is a square matrix such that $A^m=0$ where m is a +ve integer then A is called nilpotent matrix.

- If m is least positive integer such that $A^m = 0$ then A is called nilpotent of index m

Involuntary : If A is a square matrix such that $A^2 = I$ then A is called involuntary matrix.

Orthogonal Matrix : A square matrix A is said to be orthogonal if $AA^1 = A^1A = I$

- If A, B are orthogonal matrices, each of order n then AB and BA are orthogonal matrices.
- Prove that the inverse of an orthogonal matrix is orthogonal and its transpose is also orthogonal.

Conjugate of a matrix: If the elements of a matrix A are replaced by their conjugates then the resulting matrix is defined as the conjugate of the given matrix. We denote it with $\overline{A}$

The transpose of the conjugate of a square matrix:

If A is a square matrix and its conjugate is $\overline{A}$, then the transpose of $\overline{A}$ is $\left(\overline{A}\right)^T$.

It can be easily seen that $\left(\overline{A}\right)^T = \overline{A^T}$. It is denoted by A^θ

$$A^\theta = \left(\overline{A}\right)^T = \overline{A^T}$$

Note: If A^θ and B^θ be the transposed conjugates of A and B respectively, then

i) $\left(A^\theta\right)^\theta = A$ ii) $(KA)^\theta = \overline{K}A^\theta$ iii) $(AB)^\theta = B^\theta A^\theta$

Hermitian matrix:

A square matrix A such that $\overline{A} = A^T$ (or) $\left(\overline{A}\right)^T$ =A is called a Hemitian matrix.

ie if A^θ =A then A is called Hermitian.

1) The element of the principal diagonal of a Hermitian matrix must be real.

Skew-Hermitian matrix

A square matrix A such that A^θ = -A is called a Skew-Hermitian matrix

1) The elements of the leading diagonal must be zero (or) all are purely imaginary

Unitary matrix:

A square matrix A such that $A^\theta = A^{-1}$

i.e $A^\theta A$=$A\,A^\theta$ =I

If $A^\theta A$=I then A is called Unitary matrix

1 The Eigen values of a Hermition matrix are real.

2 The Eigen values of a skew-Hermition matrix are either purely imaginary (or) Zero

3 The Eigen values of a unitary matrix have absolute value l.

4 The characteristic root of an orthogonal matrix is unit modulus.

5 The only real eigen values of unitary matrix and orthogonal matrix can be $\pm$ 1

6 The transpose of a unitary matrix is unitary.

7 The inverse of a unitary matrix is unitary.

Minors and cofactors of a square matrix

Let A $=[a_{ij}]_{n\times n}$ be a square matrix when form A the elements of i^{th} row and j^{th} column are deleted, the determinant of (n-1) rowed matrix is called the minor of a_{ij} of A and is denoted by $|M_{ij}|$

The signed minor $(-1)^{i+j}|M_{ij}|$ is called the cofactor of a_{ij} and is denoted by A_{ij}.

1: If A is a square matrix of order n then $|KA| = K^n|A|$, where k is a scalar.

2: If A is a square matrix of order n, then $|A| = |A^T|$

3: If A and B be two square matrices of the same order, then $|AB| = |A|\,|B|$

Inverse of a Matrix: let A be any square matrix, then a matrix B, if exists such that AB = BA =I then B is called inverse of A and is denoted by A^{-1}.

Adjoint of a matrix:

Let A be a square matrix of order n. The transpose of the matrix got from A by replacing the elements of A by the corresponding co-factors is called the adjoint of A and is denoted by adj A. - For any scalar k, adj(kA) = k^{n-1} adj A

Singular and Non-singular Matrices:

A square matrix A is said to be singular if $|A| = 0$. If $|A| \neq 0$ then A is said to be non-singular.

Rank of a Matrix:

Let A be mxn matrix. If A is a null matrix, we define its rank to be '0'. Let A be a non-null matrix, we say that *r* is the rank of A if

(i) Every (r+1)th order minor of A is '0' (zero) &

(ii) At least one *r*th order minor of A is not zero.

it is denoted by $\rho(A)$.

- if A is a matrix of order mxn then Rank of A $\leq$ min(m,n)
- if $\rho(A) = r$ then every minor of A of order r+1, or more is zero.
- Rank of the Identity matrix I_n is n.
- If A is a matrix of order *n* and A is non-singular then $\rho(A) = n$

*** Elementary Transformations on a Matrix:**

i). Interchange of *i*th row and *j*th row is denoted by $R_i \leftrightarrow R_j$

(ii). If *i* th row is multiplied with a non-zero constant K then it is denoted by $R_i \rightarrow K R_i$

(iii). If all the elements of i^{th} row are multiplied with K and added to the corresponding elements of jth row then it is denoted by $R_j \to R_j + KR_i$

1. The corresponding column transformations will be denoted by $C_i \leftrightarrow C_j$, $C_i \to K\,C_j$ $C_j \to C_j + KC_i$

2. The elementary operations on a matrix do not change its rank.

Echelon form of a matrix:

A matrix is said to be in Echelon form, if

(i). Zero rows, if any exists, they should be below the non-zero rows.

(ii). The first non-zero entry in each non-zero row is equal to '1'.

(iii). The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.

Note : 1. The number of non-zero rows in the row echelon form of A is the rank of 'A'.

Normal Form:

Every *mxn* matrix of rank *r* can be reduced by a finite number of elementary transformations to one of the forms $[I_r]\begin{bmatrix} I_r \\ 0 \end{bmatrix}[I_r \quad 0]\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, where I_r is the *r* – rowed identity matrix.

In all the above four forms the rank of the matrix is *r*.

The inverse of a matrix by elementary Transformations: **(Gauss – Jordan method)**

1. suppose A is a non-singular matrix of order 'n' then we write $A = I_n A$
2. Now we apply elementary row-operations only to the matrix A and the pre-factor I_n of the R.H.S
3. We will do this till we get $I_n = BA$ then obviously B is the inverse of A.

System of linear simultaneous equations:

Linear Equation:

Consider the system of *m* linear equations in *n* unknowns $x_1, x_2, \ldots.. x_n$ as given below

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots.. + \ldots. a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots.. + \ldots. a_{2n}x_n = b_2 \\ ------ ---------------- \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots.. + \ldots. a_{mn}x_n = b_m \end{array}\right\} \ldots\ldots\ldots\ldots\ldots(1)$$

where a_{ij}'s and b_1, b_2---b_m are constants.

*An ordered n-tuple (x_1 x_2….x_n) satisfying all the equations in (1) simultaneously is called a solution of the system (1).

Consistent: Above system (1) have at least one solution, and then the system is called **consistent**.

If (1) does not have any solution, then the system is called **inconsistent**.

The system of equations in (1) can be written in matrix form as A X = B ----(2)

Where $A = [a_{ij}]_{mxn}$ $X= (x_1,x_2.....x_n)^T$ and $B = (b_1,b_2....b_m)^T$

Note: The matrix $[A|B]$ is called the augmented matrix of the system (1).

Homogeneous system: In AX =B, if $B = \bar{0}$ then the system is called homogeneous, otherwise the system is called non-homogeneous.

Note: 1. The system $AX = \bar{0}$ is always consistent since $X = \bar{0}$ (i.e $x_1= x_2 .....x_n =0$) is always a solution of $AX = \bar{0}$.

1. This solution is called a **trivial solution** or **zero solution** of the system.

2. If $AX = \bar{0}$ has any solution other than $X = \bar{0}$ (i.e. $x \neq \bar{0}$), such a solution is called a **non-trivial** soln or non-zero solution.

Solution of AX = B using Echelon form:

step 1 Represent the system of *m* equations in *n* unknowns in augmented matrix form

step 2 The augmented matrix of the system is $[A|B]$

step 3 Reduce the above matrix into echelon form

The system Ax = B is

(a) consistent if $\rho(A) = \rho[A/B]$

i). $\rho(A) = \rho[A/B]= n$ then the system will have unique solution.

ii). $\rho(A) = \rho[A/B] < n$ then there are infinite no of solutions.

.

(b) inconsistent if $\rho(A) \neq \rho[A/B]$ (i.e. the system has no solution).

Consistency of system of Homogeneous linear equations:

A homogeneous system is of the form $AX = \bar{0}$

- The number of linearly independent solutions of the linear system Ax = 0 is (n-r) , r being the rank of the matrix A and n being the number of variables.
- if A is a non-singular matrix then the linear system $Ax = \bar{0}$ has only the zero solution.
- The system Ax =0 possesses a non-zero soln. if and only if A is a singular matrix.

Working rule for finding the solutions of the equation $Ax = \bar{0}$

(i). Rank of A = No. of unknowns i.e $r = n$ then the given system has only zero solution.

(ii). Rank of A < No of unknowns ($r<n$) then the system has infinite no. of solutions.

Note: If $Ax = \bar{0}$ has more unknowns than equations the system always has infinite solutions.

UNIT II : Eigen Values & Eigen Vectors

Let A= [a_{ij}] be an n x n matrix, I be an identity matrix of order n x n and'λ'be a scalar then

(a) A-λI is called characteristic matrix (b) $|A-\lambda I|$ is called characteristic polynomial,

(c) $|A-\lambda I|=0$ is called characteristic equation, (d) roots of the char. eq. $|A-\lambda I| = 0$ are called characteristic roots or eigen values.

A non-zero vector X is said to be a Characteristic Vector of A if there exists a scalar λ such that AX=λ X we say 'λ' is the eigen value (or) characteristic root of 'A'.

Let A = [a_{ij}] be a nxn matrix. Let X be an eigen vector of A corresponding to the eigen value λ.

Then by definition $AX = \lambda X$.

⇨ $AX = \lambda IX$

⇨ $AX - \lambda IX = 0$

⇨ $(A-\lambda I)X = 0$ ------- (1)

This is a homogeneous system of *n* equations in *n* unknowns.

Properties of Eigen Values:

1 The sum of the eigen values of a square matrix is equal to its trace and product of the eigen values is

equal to its determinant.

2 If λ is an eigen value of A corresponding to the eigen vector X, then λ^n is eigen value A^n corresponding to the eigen vector X.

3 A Square matrix A and its transpose A^T have the same eigen values.

4 If A and B are n-rowed square matrices and A is invertible show that $A^{-1}B$ and BA^{-1} have same eigen values.

5 If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigen values of a matrix A then $k\lambda_1, k\lambda_2, \ldots k\lambda_n$ are the eigen value of the matrix KA, where K is a non-zero scalar.

6 If λ is an eigen value of the matrix. Then $\lambda \pm K$ is an eigen value of the matrix $A \pm KI$

7 If λ is an eigen value of a non-singular matrix A corresponding to the eigen vector then λ^{-1} *is an eigne vector of* A^{-1} *and corresponding eigen vectgor X itself.*

8 If λ is an eigen value of a non − singular matrix A, then $\frac{|A|}{\lambda}$ is an eigen value of the matrix Adj A

9 If λ is an eigen value of an orthogonal matrix then $\frac{1}{\lambda}$ is also an eigen value.

10 If λ is eigen value of A then prove that the eigen value of $B = a_0A^2+a_1A+a_2I$ is $a_0\lambda^2+a_1\lambda+a_2$

11 If A and B are square matrices such that A is non-singular, then $A^{-1}B$ and BA^{-1} have the same eigen values.

12 If A and B are non-singular matrices of the same order, then AB and BA have the same eigen values.

13 The eigen values of a triangular matrix are just the diagonal elements of the matrix.

14 The eigen values of a diagonal matrix are just the diagonal elements of the matrix.

15 The eigen values of a real symmetric matrix are always real

Diagonalization of a matrix:

If a square matrix A of order *n* has *n* linearly independent eigen vectors $(X_1,X_2\ldots X_n)$ corresponding to the *n* eigen values $\lambda_1,\lambda_2\ldots\lambda_n$ resp. then a matrix P can be found such that $P^{-1}AP$ is a diagonal matrix.

Modal and Spectral matrices:

The matrix P in the above result which diagnolise the square matrix A is called **modal matrix** of A and the resulting diagonal matrix D is known as **spectral matrix**.

Calculation of powers of a matrix:

In general $D^n = P^{-1}A^nP$

To obtain A^n, Premultiply (1) by P and post multiply by P^{-1}

Then $PD^nP^{-1} = A^n$

Cayley Hamilton Theorm: Every square matrix satisfies its own characteristic equation.

Quadratic Forms

Definition: A homogeneous expression of degree two in any number of variables is called Quadratic form.

1. $3x^2+5xy-2y^2$ is a quadratic form in two variables x and y.
2. $2x^2+3y^2-4z^2+2xy-3yz+5zx$ Is a quadratic form in three variables x,y and z.

An expression of the form Q= $X^T AX = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$ where a_{ij}'s are constants is called a quadratic form in *n* variables $x_1, x_2.........x_n$ .if the constants a_{ij}'s are real numbers it is called a real quadratic form.

Rank of a quadratic form:

Let $X^T AX$ be a quadratic form over R. The rank *r* of A is called the rank of quadratic form. If $r < n$ (order of A) or $|A| = 0$ or A is singular then quadratic form is called singular otherwise non-singular.

<u>Canonical form (or) Normal form (or) sum of squares form of a quadratic form:</u>

Let $X^T AX$ be a quadratic form in n variables. Then there exists a real non-singular linear transforms X=PY which transformation $X^T AX$ to the form $Y^T DY = \lambda_1 y_1^2 + \lambda_2 y_2^2 + ...\lambda_n y_n^2$ under the transformation X= PY, then $Y^T DY$ is called the canonical form of $X^T AX$ .Here D=diag $[\lambda_1, \lambda_2........\lambda_n]$.

<u>Index of a real quadratic form:</u>

When the quadratic form $X^T AX$ is reduced to the canonical form, it will contain only r terms, if the rank of A is *r*. Thus, the number non-zero terms in the canonical form is **rank** (*r*)

- The number of positive terms in a normal form of quadratic form is called the **index** (*s*) of the quadratic form.
- The number of negative terms in a normal form of quadratic form is *r – s*.
- If *r* is the rank of a quadratic form and *s* is the number of positive terms in its normal form, then the difference between the number of positive terms and the number of negative terms i.e., $s - (r - s) = 2s - r$ is called the **signature** of the quadratic form.

<u>Nature of Quadratic forms:-</u>

The quadratic form $X^T AX$ in n variables is said to be

(i) <u>Positive definite:-</u> If $r = n$ and $s = n(or)$ if all the eigen values of A are positive

(ii) <u>Negative definite:</u>- If $r = n$ and $s = 0$ (or)if all the eigen values of A are negative

(iii) <u>Positive semi definite:-</u> If $r < n$ and $s = r(or)$ if all the eigen of $A \geq 0$ and atleast one eigen value is zero

(iv) <u>Negative semi definite:-</u> If $r < n$ and $s = 0(or)$ if all the eigen of $A \leq 0$ and atleast one eigen value is zero

(v) <u>Indefinite:-</u> In all other cases

Sylvester's Law of Inertia:-

The signature of quadratic form is invariant for all normal reductions.

Reduction to Normal form by Orthogonal Transformation:-

If in the transformation $X = PY$, P is an Orthogonal matrix and if $X = PY$ transforms the quadratic form Q to the canonical form then Q is said to be reduces to canonical form by an Orthogonal transformation.

working rule:

1. Write matrix A of the quadratic form.
2. Find the eigen values of A say $\lambda_1, \lambda_2, ...\lambda_n$.
3. Find the corresponding eigen vectors say $X_1, X_2, ..., X_n$ which are pair wise orthogonal.
4. Normalise these vectors as $e_i = \frac{X_i}{\|X_i\|}$
5. Form the matrix P containing the normalized eigen vectors of A ie P=[e_1 e_2......e_n].
6. Find the diagonal matrix D as D=P^TAP where D= diag[$\lambda_1, \lambda_2, ...\lambda_n$]
7. The required Canonical form is Y^TDY=$\lambda_1 y_1^2 + \lambda_2 y_2^2 + ... + \lambda_n y_n^2$.

UNIT – III : Calculus

MEAN VALUE THEOREMS

I Rolle's Theorem:

Let f:[a, b]→R be a function such that

(i). it is continuous on [a, b]

(ii) it is differentiable on (a, b)

(iii) f(a) = f(b). then there exists at least one point 'c' in open (a, b) such that $f'(c) = 0$.

II Lagrange's Mean value Theorem

Let f:[a, b]→R be a function such that

(i) it is continuous on [a, b]

(ii) it is differentiable on (a, b)

then there exists at least one point 'c' in open (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

III. Cauchy's Mean Value Theorem

If f: [a,b] →R, g:[a,b] →R are two functions such that

(i) f,g are continuous on [a,b] (ii) f,g are diff on (a,b) (iii) $g'(x) \neq 0 \ \forall$ x then

$\exists\, c \in (a,b)$ such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$.

UNIT-IV : Multi Variable Calculus(Partial differentiation and Applications)

Limits & Continuity

Properties on limits.

1. If $\lim_{x\to a} f(x)$ exits then, it is unique.
2. $\lim_{x\to a} f(x) = \lim_{x\to 0} f(a-x) = \lim_{x\to 0} f(a+x)$.
3. If f(x) is constant function, i.e f(x) = c then $\lim_{x\to a} f(x)$ = c.

Continuity:

A function *f* is said to be continuous at x=a if $\lim_{x\to a} f(x) = f(a)$ i.e $\lim_{x\to a+} f(x) = \lim_{x\to a+} f(x) = f(a)$

1. An identity function *f*(x) = x is continuous on R.
2. Every constant function is f(x) = k is continuous on R.
3. Every polynomial function is continuous on R.
4. f(x) = logx , f(x) = $\sqrt{x}$ are cont. on R^+.
5. f(x)= e^x, f(x)= e^{-x}, f(x) = sinx, f(x) = cosx are continuous on R.
6. f(x) = tanx is not cont. at x = (2n+1)π/2
7. A function is said to be continuous on a set S if it is continuous at every point of S.

Derivatives

A function *f* is said to be differentiable at x=c if $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists, and it is denoted by $f'(c)$.

1. Every differentiable function is continuous but a continuous function need not be differentiable.
2. A function is said to be differentiable on a set S if it is differentiable at every point of S.

Jacobian (J) :

Def 1: Let u= u(x , y) , v = v(x , y) are two functions of the independent variables x , y. The jacobian of u , v w.r.t x, y is given by

$$J\left(\frac{u,v}{x,y}\right)=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

Note : $\mathrm{J}\left(\frac{u,v}{x,y}\right)\times \mathrm{J}\left(\frac{x,y}{u,v}\right)=1$

Def 2: Similarly of U = u(x, y , z) , V = v (x, y , z) , W = w(x, y , z)

Then the Jacobian of u , v , w w.r.to x , y , z is given by

$$J\left(\frac{u,v,w}{x,y,z}\right)=\frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

Functional Dependence

Two functions u and v are functionally dependent if their Jacobian is zero i.e.,

$$J\left(\frac{u,v}{x,y}\right)=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}=0.$$

If the Jacobian of *u*, *v* is not equal to zero then those functions *u*, *v* are functionally independent.

Maxima & Minima for functions of two Variables:

Let f(x,y) be a function of two variables then f(x,y) is said to have

(i) maximum at (a, b) if f(a,b) is the maximum in a neighborhood of (a,b) and f(a, b) is called maximum value.

(ii) minimum at (a, b) if f(a,b) is the minimum in a neighborhood of (a,b) and f(a, b) is called minimum value.

Working Rule:

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ and equate each to zero. Solve these equations for x & y we get the pair of values (a_1,b_1) (a_2,b_2) (a_3,b_3) ………………

2. First consider the point (a_1,b_1)

   Find $l=\frac{\partial^2 f}{\partial x^2}, m=\frac{\partial^2 f}{\partial x\,\partial y}$, n = $\frac{\partial^2 f}{\partial y^2}$ at the point (a_1,b_1)

i) IF $ln-m^2>0$ and $l<0$ at (a_1,b_1) then f(x ,y) is maximum at (a_1,b_1) and maximum value is $f(a_1,b_1)$.

ii) IF $ln-m^2>0$ and $l>0$ at (a_1,b_1) then f(x ,y) is minimum at (a_1,b_1) and minimum value is $f(a_1,b_1)$.

iii) IF $l n - m^2 < 0$ and at (a_1,b_1) then f(x ,y) is neither maximum nor minimum at (a_1,b_1). In this case (a_1,b_1) is saddle point.

iv) IF $l n - m^2 = 0$ and at (a_1,b_1) , no conclusion can be drawn about maximum or minimum and needs further investigation.

Similarly we do this for other stationary points.

Extremum: A function which have a maximum or minimum or both is called 'extremum'.

Extreme value: The maximum value or minimum value or both of a function is Extreme value.

Stationary points:- To get stationary points we solve the equations $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ i.e the pairs (a_1, b_1), (a_2, b_2) ………….. are called Stationary points.

Maxima & Minima for a function with constant condition: Lagrangian Method

Suppose we have to maximize or minimize the function f(x , y , z) ------------(1)

subject to the condition $\varphi(x, y, z) = 0$ ------------- (2)

Form the Lagrange's function $F(x, y, z) = f(x, y, z) + \lambda\, \varphi(x, y, z)$

where λ is called Lagrange's constant.

1. $\frac{\partial F}{\partial x} = 0 \Rightarrow \frac{\partial f}{\partial x} + \lambda \frac{\partial \phi}{\partial x} = 0$ --------------- (3)

   $\frac{\partial F}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial y} + \lambda \frac{\partial \phi}{\partial y} = 0$ --------------- (4)

   $\frac{\partial F}{\partial z} = 0 \Rightarrow \frac{\partial f}{\partial z} + \lambda \frac{\partial \phi}{\partial z} = 0$ --------------- (5)

2. Solving the equations (2) (3) (4) & (5) we get the stationary point (x, y, z).
3. Substitute the value of x , y , z in equation (1) we get the extremum.

UNIT V : FUNCTION OF SEVERAL VARIABLES

Double Integral : If x,y are two variables, R is a region in xy-plane and f(x, y) is function defined over R then the double integral of f(x, y) over the region R is defined by

$\iint_R f(x, y)\,dxdy$.

Type I $\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy$ (The limits of x and y are constants)

Type II $\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} f(x,y)dxdy$ (The limits of x are constants and limits of y are functions of x)

Type III $\int_{y=c}^{d}\int_{x=f_1(y)}^{f_2(y)} f(x,y)dxdy$ (The limits of x are functions of y and limits of y are constants).

Change of order of integration :

consider $\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} f(x,y)dxdy$ {limits of x (which are constants) and limits of y (which are functions of x)}

which has to be evaluated by change of integral.

step 1. Write the given limits of x and y

step 2. Draw the graph based on limits of y

step 3. Find point of intersections of the curves, if any

step 4. Based on them, find new limits of y (which are constants) and limits of x (which are functions of y)

The change integral is of the form

$$\int_{y=c}^{d}\int_{x=f_1(y)}^{f_2(y)} f(x,y)dxdy$$

step 5. Finally, solve the above integral.

In general, in plane polar coordinates,

$$\iint_D f\left(x,y\right)dA = \int_{\alpha}^{\beta}\int_{g(\theta)}^{h(\theta)} f\left(r\cos\theta, r\sin\theta\right)r\,dr\,d\theta$$

<u>Triple Integrals</u>

The concepts for double integrals (surfaces) extend naturally to triple integrals (volumes).
The element of volume, in terms of the Cartesian coordinate system (x, y, z) and another orthogonal coordinate system (u, v, w), is

$$dV = dx\,dy\,dz = \frac{\partial(x,y,z)}{\partial(u,v,w)}\,du\,dv\,dw$$

and

$$\iiint_V f(x,y,z)\,dV = \int_{w_1}^{w_2}\int_{v_1(w)}^{v_2(w)}\int_{u_1(v,w)}^{u_2(v,w)} f\left(x(u,v,w),y(u,v,w),z(u,v,w)\right)\frac{\partial(x,y,z)}{\partial(u,v,w)}\,du\,dv\,dw$$

The most common choices for non-Cartesian coordinate systems in $\mathbb{R}^3$ are:

Cylindrical Polar Coordinates:

$$x = r\cos\phi$$
$$y = r\sin\phi$$
$$z = z$$

for which the differential volume is

$$dV = \frac{\partial(x,y,z)}{\partial(r,\phi,z)}\,dr\,d\phi\,dz = r\,dr\,d\phi\,dz$$

Spherical Polar Coordinates:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

for which the differential volume is

$$dV = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}\,dr\,d\theta\,d\phi = r^2\sin\theta\,dr\,d\theta\,d\phi$$

Let V be a certain solid occupying the domain V given in the Cartesian coordinate system and let there in V be distributed mass with the volume mass density $\gamma = \gamma(x,y,z)$. Then the product $\gamma(x,y,z)dxdydz$ is differential mass element located at the point (x,y,z). Using it in the corresponding physical characteristics, after integration over V one obtain formulas given below.

$$m = \iiint_V \gamma(x,y,z)\,dx\,dy\,dz \qquad (24)$$

is mass of a solid V ; If

$$M_{zx} = \iiint_V y\gamma(x,y,z)\,dx\,dy\,dz,$$

$$M_{yz} = \iiint_V x\gamma(x,y,z)\,dx\,dy\,dz,$$

$$M_{xy} = \iiint_V z\gamma(x,y,z)\,dx\,dy\,dz$$

are static moments of a solid with respect to the coordinate planes $Ozx,\ Oyz,\ Oxy$ correspondingly.

$$x_c = \frac{M_{yz}}{m},\ y_c = \frac{M_{zx}}{m},\ z_c = \frac{M_{xy}}{m}$$

are coordinates of the center of mass of a solid.

UNIT WISE QUESTION BANK

UNIT – I Matrices

I.Short Answer questions

1. Find the value of k if the rank of the matrix is 2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 6 \\ -1 & 0 & 3 \end{bmatrix}$$

2. Define Hermitian matrix with examples.
3. Prove that every square matrix can be expressed as a sum of symmetric and skew-symmetric matrix.

II. Long answer Questions

1. Define the rank of a matrix and Reduce the matrix to Echelon form hence find the

rank of $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$.

2. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ into Echelon form and hence find its rank.

3. Reduce the matrix to normal form and hence find its rank A= $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

4. Determine for what values of λ & μ the simultaneous equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have

   i) No Solution ii) Unique solution iii) Many solutions.

5. Determine for what values of k, the equations $x+y+z=1,\ 4x+y+10z=k^2,\ 2x+y+4z=k$ have a solution and then solve them completely.

6. Prove that the following set of equations is consistent and solve them:

$3x+3y+2z=1,\ x+2y=4,\ 10y+3z=-2, 2x-3y-z=5$

7. Find the inverse of a matrix by Gauss-Jordan Method $\begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

8. Using Gauss – Jordan method, solve the system: $x+y+z=10,\ 3x+2y+3z=18, x+4y+9z=16$

9. Solve the equations $2x_1+x_2+x_3=10,\ \ 3x_1+2x_2+3x_3=18,\ x_1+4x_2+9x_3=16$ using Gauss elimination method.

10. Solve the equations $10x_1+x_2+x_3=12,\ \ x_1+10x_2-x_3=10,\ x_1-2x_2+10x_3=9$ by Gauss-Jordan method.

11. Define Hermitian and Skew-Hermitian Matrix. Show that $A=\begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is

Skew- Hermitian Matrix.

III. Each question carries ½ mark.

1) The index of the matrix $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ is ____

2) The rank of a matrix in Echelon form is equal to ___________________________________

3) If r<n, where r is the rank of the coefficient matrix and n is the no. of unknown of the system Ax=0, then it possesses ____

4) The system Ax=B has an infinite no. of solutions if
   (a) P(A)=P([A:B]) = r and r < n   (b) P(A)=P([A:B]) = r and r = n
   (c) P(A)=P([A:B]) = r and r > n   (d) none

5) The number of linearly independent solutions of AX=0 is __________________ where P(A) =*r* and *n* is the no. of unknowns
   (a) n-(r-1)   (b) n-(r+1)   (c) n-r   (d) none

6.) If $\overline{A}=A^T$ then A is called
   a) Hermitian Matrix   b) Skew-Hermitian Matrix
   c) Symmetric Matrix   d) None

7) The rank of null matrix is
   a) 0   b) 1   c)2   d) none

8) The rank of a matrix in Echelon form is equal to
a) No. of non zero rows b) No. of non zero columns
c) Determinant of the matrix d) None

9) The system Ax = B where $B \neq 0$ is called
a) Homogeneous system b) Non-Homogeneous system
c) Linear system d) None

10) Matrix A is Symmetric, if
a) $A = A^T$ b) $A = -A^T$ c) $AA^T = I$ d) None

UNIT-II : Eigen values and Eigen vectors

I. Short Answer questions

1. Write the matrix form of the quadratic form $x_1^2 - 16x_1x_2 + 4x_2^2$
2. Write any three properties of eigen values and eigen vectors.
3. Prove that eigen values of a Hermitian matrix are purely imaginary or zero.

II. Long answer Questions

1. If λ is an Eigen value of A then prove that λ^n is an Eigen value of A^n.
2. Find the Eigen values and corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3. State Cayley - Hamilton theorem. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A.
4. Find the Eigen values and corresponding Eigen vectors of the matrices

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

5. Show that the matrix A = $\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation
6. Diagonalize the matrix A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$
7. Show that Two Similar Matrices have Same Eigen Values
8. Find an Orthogonal Matrix that will Diagonalize the real Symmetric Matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

9. Diagonalize the matrix A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

III. Each question carries ½ mark.

1. Product of Eigen values of a matrix A is equal to
   a) Trace of A b) Determinant of A
   c) Zero d) None

2. If A and B are similar matrices then A and B have
   a) Same Characteristic roots b) Different Characteristic roots
   c) No characteristic roots d) None

3. If A is a triangular matrix then the Eigen values of A are
   a) Diagonal elements b) First row elements
   c) First column elements d) None

4. If the Eigen values of n x n matrix are all distinct then it is similar to
   a) Diagonal Matrix b) Square matrix
   c) Rectangular matrix d) None

5. If λ is an eigen value of A, then the eigen value of adj A is
   a) λ b) $\frac{1}{\lambda}$ c) $\frac{|A|}{\lambda}$ d) None

UNIT – III: Calculus

I. Short Answer questions

1. Verify Rolle's Theorem for $f(x) = x^2 - 2x - 3$ in (-3, 1).
2. Obtain the Taylor's Series expansion of e^x about $x = -a$
3. Find Maclaurin's theorem with Lagrange's form of Remainder for $f(x) = \cos x$
4. Define Beta function.
5. Explain relation between Beta and Gamma functions.

II. Long answer Questions

1.Verify Rolle's theorem for f(x) = x(x+3)e$^{\frac{-x}{2}}$ in the interval [-3,0].

2. If a < b, Prove that $\frac{b-a}{1+b^2} <$ tan $^{-1}$b - tan $^{-1}$a $< \frac{b-a}{1+a^2}$ using Lagrange's mean value theorem. Deduce the following $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$

3.Verify the Lagrange's Mean Value theorem for $f(x) = x^2$ in (1,5)

4.Verify Cauchy's mean value theorem for f(x) = sin x , g(x) = cos x on [0 , $\frac{\pi}{2}$]

5.Obtain the Taylors series expansion of sin x in powers of x - $\frac{\pi}{4}$

6.Write Taylors series for f(x) = $(1 - x)^{\frac{5}{2}}$ with Lagrange's form of remainder up to 3 terms in the interval [0 , 1]

7. If $f(x) = \sin^{-1} x$ and 0<a<b<1, prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$

8. Derive relation between Beta and Gamma function ?

III. Each question carries ½ mark.

1. Mean value theorem for $f(x) = |x|$ in [-1,1]
   a. Not applicable due to discontinuity
   b. Not applicable due to Non- Differentiability at x=0
   c. Applicable
   d. None
2. Mean Value Theorem for $f(x) = \tan x$ in $[0, \pi]$
   a. Not applicable due to discontinuity at $x = \frac{\pi}{2}$
   b. Not applicable due to Non- Differentiability at $x = \frac{\pi}{2}$
   c. Applicable   d. None
3. Generalized mean value theorem for $f(x) = \sec x$ in $(0, 2\pi)$ is
   a. Not applicable due to discontinuity
   b. Not applicable due to Non- Differentiability
   c. Applicable and $c = \pi$
   d. Applicable
4. By Rolle's Theorem if f(a)=f(b) for a<c<b then
   a. $f'(c) = 0$ b. $f'(c) \neq 0$ c. $f'(c) < 0$ d. None
5. By Lagrange's Mean value theorem $f'(c) =$
   a. $\frac{f(b) - f(a)}{b - a}$ b. $\frac{f(b) + f(a)}{b + a}$ c. $\frac{f(b) - f(a)}{b + a}$ d. None

6. If $f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + ... f^{(n)}(0)\frac{x^n}{n!}$ then the series is called

a. Maclaurin's series   b.Taylor's series   c. Cauchy's series   d. None

UNIT-IV: Multi Variable Calculus(Partial differentiation and Applications)

I. Short Answer questions

1) Evaluate $\underset{\substack{x\to 1\\ x\to 2}}{Lim} \dfrac{2x^2 y}{x^2+y^2+1}$

2) If u=$\dfrac{1}{\sqrt{x^2+y^2+z^2}}$ prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=0$ where $x^2+y^2+z^2 \neq 0$

3) If u = $\tan^{-1}$ ($\dfrac{2xy}{x^2-y^2}$) prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$

4) If r=$x^2+y^2+z^2$ and u r^m then prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}$=m(m+1)$r^{m-2}$

5) If x=rcosθ,y=rsinθ find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ and $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ show that $\dfrac{\partial(x,y)}{\partial(r,\theta)}\dfrac{\partial(r,\theta)}{\partial(x,y)}$=1

II. Long answer Questions

1. If x+y+z=4,y+z=uv,z=uvw evaluate $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$
2. Show that the functions u=xy+yz+zx,v=$x^2+y^2+z^2$ and w=x+y+z are functionally related . find the relation between theorem.
3. Find the maximum and minimum values of f(x,y)=$x^3+3xy^2-3x^2-3y^2+4$
4. A rectangular box open at the top is to here volume of 32 cubic ft find dimensions of the box requires least material for its construction

III. Each question carries ½ mark.

1. If J represents Jacobian then JJ^I = [ ]

   a) 0 b) 1 c) -1 d) None

2. If u(x,y) and v(x,y) are functionally dependent then $J\left(\dfrac{u,v}{x,y}\right)$= [ ]

   a) 1 b) 0 c) -1 d) ½

3. The stationary points of x^3y^2(1-x-y) are [ ]

   a) (0,1) b) (-1,-1) c) (12,13) d) (1,1)

4. If f(x,y) has no maximum and no minimum at (a,b) then the point (a,b) to called

   ____

5. To find the extremes of the functions f(x,y,z) subject to the condition ∅(x,y,z) the Lagrange's function is defined as __________________

UNIT-V: Multivariable Calculus (Integration)

I.Short Answer questions

1) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+y}(x+y+z)dxdydz$

2) Evaluate $\int\int r\sin\theta\ dr\ d\theta$ over the cardroid $r=a(1-\cos\theta)$ above the initial line.

3) Change the order of integration in I = $\int_{0}^{1}\int_{x^2}^{2-x} xydxdy$ and hence evaluate the same.

4)Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\ dxdy$ by changing to polar coordinates

II. Long answer Questions

1) Evaluate $\iint(x^2+y^2)dxdy$ in the positive quadrant for which x+y<=1
2) Evaluate $\iint(z^2dxdydz$ taken over the volume bounded by $x^2+y^2=a^2, x^2+y^2=z$ and z=0.
3) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dx\ dy\ dz$

III. Each question carries ½ mark.

1. $\int_{0}^{1}\int_{0}^{1}(x^2+y^2)dxdy$ = ( )

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{1}$

2. $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}}\frac{dxdydz}{\sqrt{1^2-x^2-y^2-z^2}}$ = ______________________________

Change the order of integration in $\int_{0}^{\infty}\int_{x}^{\infty}\frac{e^{-y}}{y}dxdy$.