BVRIT HYDERABAD
College of Engineering for Women

Approved by AICTE and Affiliated to JNTUH, Hyderabad
Accredited by NBA & NAAC (A Grade)
Rajiv Gandhi Nagar, Bachupally, HYDERABAD – 500090
Telangana, India

COURSE CONTENT	
Department	Basic Sciences and Humanities
Year/Semester	I B.Tech. / II Semester
Subject	Ordinary differential equations & Vector calculus
Regulation	R22

VISHNU
UNIVERSAL LEARNING

VISION

To emerge as the best among the institutes of technology and research in the country dedicated to the cause of promoting quality technical education.

MISSION

At BVRITH, we strive to

- Achieve academic excellence through innovative learning practices.
- Enhance intellectual ability and technical competency for a successful career.
- Encourage research and innovation.
- Nurture students towards holistic development with emphasis on leadership skills, life skills and human values.

ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS
(Common to ECE, EEE, IT, CSE)
B.Tech. I Year II Sem

Pre-requisites: Mathematical Knowledge at pre-university level
Course Objectives: To learn

- Methods of solving the differential equations of first and higher order.
- Concept, properties of Laplace transforms
- Solving ordinary differential equations using Laplace transforms techniques.
- The physical quantities involved in engineering field related to vector valued functions
- The basic properties of vector valued functions and their applications to line, surface and volume integrals

Course outcomes: After learning the contents of this paper the student must be able to

- Identify whether the given differential equation of first order is exact or not
- Solve higher differential equation and apply the concept of differential equation to real world
- problems.
- Use the Laplace transforms techniques for solving ODE's.
- Evaluate the line, surface and volume integrals and converting them from one to another

UNIT-I: First Order ODE (8 L)

Exact differential equations, Equations reducible to exact differential equations, linear and Bernoulli's equations, Orthogonal Trajectories (only in Cartesian Coordinates). Applications: Newton's law of cooling, Law of natural growth and decay.

UNIT-II: Ordinary Differential Equations of Higher Order (10 L)
Second and Higher order linear differential equations with constant coefficients, Non-Homogeneous terms of the type $\sin ax$, $\cos ax$, e^{ax}, polynomials in x, $e^{ax}V(x)$ and $x\,V(x)$, Method of variation of parameters. Equations reducible to linear ODE with constant Coefficients, Legendre's equation and Cauchy-Euler equation. Applications : Electric Circuits.

UNIT-III: Laplace transforms (10 L)
Laplace Transforms: Laplace Transform of standard functions, First shifting theorem, Second shifting theorem, Unit step function, Dirac delta function, Laplace transforms of functions when they are multiplied and divided by 't', Laplace transforms of derivatives and integrals of function, Evaluation of integrals by Laplace transforms, Laplace transform of periodic functions, Inverse Laplace transform by different methods, convolution theorem (without proof). Applications: solving Initial value problems by Laplace Transform method.

UNIT-IV: Vector Differentiation (10 L)
Vector point functions and scalar point functions, Gradient, Divergence and Curl, Directional derivatives, Tangent plane and normal line, Vector Identities, Scalar potential functions, Solenoidal and Irrotational vectors.

UNIT-V: Vector Integration (10 L)

Line, Surface and Volume Integrals, Theorems of Green, Gauss and Stokes (without proofs) and their applications.

TEXT BOOKS:

1. B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010 R22 B.Tech. CSE (AI and ML) Syllabus JNTU Hyderabad
2. R.K. Jain and S.R.K. Iyengar, Advanced Engineering Mathematics, Narosa Publications, 5th Edition, 2016.

REFERENCE BOOKS:

1. Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
2. G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9th Edition, Pearson, Reprint, 2002.
3. H. K. Dass and Er. Rajnish Verma, Higher Engineering Mathematics, S Chand and Company Limited, New Delhi.
4. N.P. Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications, Reprint, 2008.

Course Outcomes

ODE & VC	Course Outcomes	Bloom's Taxonomy
C121.1	Solve geometrical and physical problems using first order and first degree differential equations	Analyze
C121.2	Solve higher order linear differential equations with constant coefficients	Apply
C121.3	Evaluate Laplace and inverse Laplace transforms of various functions	Apply
C121.4	Apply Laplace Transforms to solve ordinary differential equations	Apply
C121.5	Analyze the properties of Differential Operators	Analyze
C121.6	Evaluate the line, surface, and volume integrals using their inter-relationships	Apply

BVRIT HYDERABAD
College of Engineering for Women
Bachupally, Hyderabad – 500090
Department of Basic Science and Humanites
B.Tech I Year **LESSON PLAN**

Course Code: **Course Title : ODE & VC**
Class: CSE/ECE/EEE/IT **Academic Year : 2022-23**

UNIT – I: First Order ODE

Exact, linear and Bernoulli's equations; Applications : Newton's law of cooling, Law of natural growth and decay; Equations not of first degree: equations solvable for p, equations solvable for y, equations solvable for x and Clairaut's type.

Session No.	Date	Topic Proposed to be Covered	Text /Reference Book	Chapter No. & Page No.	Web Resources	COs Achieved
1		Differential equations: Introduction, Order, Degree and examples.	T1	T1: 426-431 T2:8.1-8.3	1.nptel.ac.in/courses/122107037/ 2.nptel.ac.in/courses/101108047/module8/Lecture%2017.pdf	Solve ODE's by Analytical Methods
2		Exact differential equations: General form, Solution procedure, problems.	T1	T1: 440-442 T2: 8.9-8.11		
3		Non exact differential equations: 6 cases of finding Integrating Factors.	T1	T1: 442-445, T2: 8.11-8.17		
4		Non exact differential equations: finding Integrating Factors: contd..	T1	T1: 442-445, T2: 8.11-8.17		
5		Practice problems	T1	T1: 442-443, T2: 8.11-8.13		
6		Linear differential equations: General form, Solution procedure, problems.	T1	T1: 435-437 T2: 8.18-8.21		
7		Bernoulli differential equations, problems.	T1	T1: 437-439 T2: 8.21-8.24		
8		Practice Problems	T1			
9		Orthogonal Trajectories	T1	T1:455-457		
10		Orthogonal Trajectories	T1	T1:455-457		
11		Newtons Law of Cooling	T1	T1: 466-467 T2: 8.37-8.38		
12		Law of natural growth	T1			
13		Law of Decay	T1			
14		Practice Problems	T1			
15		Review of previous years question papers	T1			

		Tutorial	T1,T2			
1		**Activity- Quiz**				
UNIT –II: ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER Second and Higher order linear differential equations with constant coefficients, Non-Homogeneous terms of the type $\sin ax$, $\cos ax$, e^{ax}, polynomials in x, $e^{ax}V(x)$ and $x\,V(x)$, Equations reducible to linear ODE with constant Coefficients, Legendre's equation and Cauchy-Euler equation.						
16		Linear D. E.s of second order with constant coefficients: Introduction, solution procedure, 3 cases of Complementary function;	T1,T2	T1: 471-474, T2:9.1-9.8	**1.http://www.mathway.com** **2.** nptel.ac.in/courses/122107037/20	Model an ODE & Solve real time engineer problems.
17		Finding Particular integral when Q(x) = e^{ax}, sinax, cosax and problems.	T1,T2	T1:475-477, T2:9.9-9.16		
18		Finding Particular integral when Q(x)= x^n, Ve^{ax} and problems.	T1,T2	T1:478-486, T2:9.16-9.21		
19		Finding Particular integral when Q(x)= xV and problems.	T2	T2:9.21-9.25		
20		Problems on Complementary function and particular integral	T1,T2	T1: 486, T2:9.21-9.25		
21		Problems continued.	T1,T2	T1: 486, T2:9.21-9.25		
22		Equations reducible to linear ODE with constant Coefficients:	T1,T2	T1:490-493, T2:9.25-9.28		
23		Problems continued.	T1,T2	T1:490-493, T2:9.25-9.28		
24		Euler's or Cauchy's equation and problems.	T1,T2	T1:490-493, T2:9.25-9.28		
25		Legendre's equation and problems.	T1,T2	T1:493-495, T2:9.28-9.29		
26		Problems continued.	T1	T1:471-490,		

27		Appications: Electric Circuits	T1	T1:514-515		
28		Appications: Electric Circuits	T1	T1:514-515		
29		**Review of previous years question papers**	T1, T2	T1:471-490, T2:73-74		
30		**Tutorial**	**T1, T2**	T1:471-490, T2:73-74		
2		**Activity- Quiz**				

UNIT-III: Laplace Transforms

Laplace Transforms: Laplace Transform of standard functions, First shifting theorem, Second shifting theorem, Unit step function, Dirac delta function, Laplace transforms of functions when they are multiplied and divided by ‘t’, Laplace transforms of derivatives and integrals of function, Evaluation of integrals by Laplace transforms, Laplace transform of periodic functions, Inverse Laplace transform by different methods, convolution theorem (without proof). Applications: solving Initial value problems by Laplace Transform method

31		Basics of Laplace transforms	T1	T1:726	https://www.youtube.com/watch?v=c9NibpoQjDk https://www.youtube.com/watch?v=JzaaQxkL6Ak	Able to solve the ordinary differential equations using Laplace Transforms
32		Laplace Transform of standard functions	T1	T1:727		
33		problems	T1	T1:731-732		
34		First shifting theorem	T1	T1:728		
35		Second shifting theorem	T1	T1:756		
36		Unit step function, Dirac delta function	T1	T1:756,761		
37		Laplace transforms of functions when they are multiplied and divided by ‘t	T1	T1:735,737		
38		Laplace transforms of functions when they are multiplied and divided by ‘t	T1	T1:735,737		
39		Laplace transforms of derivatives and integrals of function	T1	T1:735		
40		Evaluation of integrals by Laplace transforms, Laplace transform of periodic functions	T1	T1:739,732		
41		Inverse Laplace transform by different methods	T1	T1:740-747		
42		Inverse Laplace transform by different methods	T1	T1:740-747		

43		Convolution theorem	T1	T1:748-750		
44		solving Initial value problems by Laplace Transform method	T1	T1:750-754		
45		solving Initial value problems by	T1	T1:750-754		
46		**Review of previous years question papers**	T1			
47		**Tutorial**	T1			
3		**Activity-**				
UNIT-IV: Vector Differentiation Vector point functions and scalar point functions. Gradient, Divergence and Curl. Directional derivatives, Tangent plane and normal line. Vector Identities. Scalar potential functions. Solenoidal and Irrotational vectors.						
48		Introduction about Vectors and their properties, applications	T1	T1:315-323	1.nptel.ac.in/courses/111106053/37 2. nptel.ac.in/courses/115101005/downloads/lectures-doc/Lecture-1.pdf	Apply knowledge of derivative to solve the problems in vector differentiation
49		Scalar point function &vector point function	T1			
50		Gradient their related properties	T1			
51		Directional derivatives	T1			
52		Tangent plane and normal line.	T1			
53		Divergence their related properties – Solenoidal vector	T1	T1:324-333		
54		Curl their related properties - irrotational vector	T1			
55		Finding Potential function	T1			
56		Laplacian operator	T1	T1:334-335		
57		**Review of previous years question papers**				
58		**Tutorial**	T1			
4		**Activity – Chart Preparation**				
UNIT-V: Vector Integration Line, Surface and Volume Integrals. Theorems of Green, Gauss and Stokes (without proofs) and their applications.						
59		Introduction to Vector integration and its applications	T1	T1:335-354	1.	. Analyze the properties
60		Line integral, work done	T1			
61		Surface integrals	T1			
62		Volume integral	T1			

63		Green's Theorem and their related problems	T1		nptel.ac.in/courses/104104086/4	of vector valued functions and their applications to line, surface & volume integrals
64		Stoke's theorem(Without proof)and their related problems	T1			
65		Gauss's Divergence Theorems (Without proof)and their related	T1			
66		**Review of previous years question papers**	**T1, T2**			
67		**Tutorial**				
5		**Activity- Mind Map**				

TEXT BOOKS:

T_1 B. S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 36th Edition, 2010

T_2 Erwin kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006

T_3 G.B. Thomas and R.L. Finney, Calculus and Analytic geometry, 9thEdition, Pearson, Reprint, 2002.

REFERENCES:

R_1. Paras Ram, Engineering Mathematics, 2nd Edition, CBS Publishes

R_2. L. Ross, Differential Equations, 3rd Ed., Wiley India, 1984.

OTHER REFERENCE BOOKS:

O_1. Engineering Mathematics-2 by T.K.V. Iyengar, B.Krishna Gandhi & Others, S.Chand , Vol-1

O_2. Engineering Mathematics – II by T.K. V. Iyengar, B. Krishna Gandhi & Others, S.Chand,Vol-2

Signature of Faculty **HOD**

SOME USEFUL FORMULAE FROM INTERMEDIATE

TRIGNOMETRIC FORMULAE:

θ	***0°***	***30°***	***45°***	***60°***	***90°***
***sin* θ**	0	1/2	1/√2	√3/2	1
***cos* θ**	1	√3/2	1/√2	1/2	0
***tan* θ**	0	1/√3	1	√3	∞

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = co\sec^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^3 x = \frac{1}{4}[3\sin x - \sin 3x] \quad \cos^3 x = \frac{1}{4}[3\cos x + \cos 3x]$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B) \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\cosh ax = \frac{e^{ax} + e^{-ax}}{2}$$

$$\sinh ax = \frac{e^{ax} - e^{-ax}}{2}$$

DIFFERENTIATION FORMULAE:

$$\frac{d}{dx}(K)=0$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}(a^x)=a^x\log a$$

$$\frac{d}{dx}(e^x)=e^x$$

$$\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}$$

$$\frac{d}{dx}(\log x)=\frac{1}{x}$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x$$

$$\frac{d}{dx}(\cot x)=-\cos ec^2 x$$

$$\frac{d}{dx}(\sec x)=\sec x\ \tan x$$

$$\frac{d}{dx}(\cos ecx)=-\cos ecx\ \cot x$$

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos ec^{-1}x)=-\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x$$
$$\frac{d}{dx}(\cosh x) = \sinh x$$
$$\frac{d}{dx}(\tanh x) = \sec h^2 x$$
$$\frac{d}{dx}(\coth x) = -\cos ech^2 x$$

$$\frac{d}{dx}(Ku) = K\frac{d}{dx}(u)$$
$$\frac{d}{dx}(u+v) = \frac{d}{dx}(u) + \frac{d}{dx}(v)$$
$$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) \; - \; u\frac{d}{dx}(v)}{v^2}$$

$$\frac{d}{dx} f[g(x)] = f'[g(x)] \times g'(x)$$

Partial Differentiation

If $U(x, y)$ is a function of two variables then

(i) partial differentiation of $U(x, y)$ wrto x means partially differentiation of $U(x, y)$ considering y as constant. It is denoted $\frac{\partial U}{\partial x}$.

(ii) partial differentiation of $U(x, y)$ wrto y means partially differentiation of $U(x, y)$ considering x as constant. It is denoted $\frac{\partial U}{\partial y}$.

INTEGRATION FORMULAE:

$$\int k \; dx = k \; x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int \log x dx = x\log|x| - x + c$$

$$\int a^x dx = \frac{a^x}{\log a} + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \cos ec^2 x \, \mathrm{dx} = -\mathrm{cotx} + \mathrm{c}$$

$$\int \sec x \tan x dx = \sec x + c$$

$$\int \cos ecx \cot x dx = -\cos ecx + c$$

$$\int \tan x dx = -\log|\cos x| + c \quad \text{or} \quad \log|\sec x| + c$$

$$\int \cot x dx = \log|\sin x| + c$$

$$\int \sec x dx = \log|\sec x + \tan x| + c \text{ or } \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$$

$$\int \cos ecx dx = \log|\cos ecx - \cot x| + c \; or \; \log\left|\tan\frac{x}{2}\right| + c$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int \tanh x dx = \log \cosh x + c$$

$$\int \coth x dx = \log \sinh x + c$$

$$\int \operatorname{sech}^2 x dx = \tanh x + c$$

$$\int \cos ec\mathrm{h}^2 x dx = -\coth x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + \mathrm{c} \; or \; -\cos^{-1} x + \mathrm{c}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \quad or - \cot^{-1} x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1} x + c \quad or \quad -\cos ec^{-1} x + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c \; or \; -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c \; or \; -\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c \; or \; -\frac{1}{a}co\sec^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\int e^{ax}\sin bx\;dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right)$$

$$\int e^{ax}\cos bx\;dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)$$

INTEGRATION BY PARTS:

Integration by parts is used in integrating product of functions of the type $f(x).g(x)$ as follows:

$$\int\left(I^{st}\,function\times II^{nd}\,function\right)dx = I^{st}\,function\int\left(II^{nd}\,function\right)dx$$
$$-\int\left(\frac{d}{dx}\left(I^{st}\,function\right)\times\int\left(II^{nd}\,function\right)dx\right)dx$$

Where the I^{st} and II^{nd} functions are decided in the order of ***ILATE***;

I: Inverse trigonometric function

L: Logarithmic function

T: Trigonometric functions

A: Algebraic functions

E: Exponential Functions

I $\int\left[f_1(x)\pm f_2(x)\right]dx=\int f_1(x)\,dx\pm\int f_2(x)\,dx$

II $\int k.f(x)\,dx=k\int f(x)\,dx$

III $\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+c, n\neq -1$

$$\int\frac{1}{ax+b}dx=\frac{\log|ax+b|}{a}+c$$

$$\int e^{ax+b}dx=\frac{e^{ax+b}}{a}+c$$

$\int\sin(ax+b)\,dx=-\frac{\cos(ax+b)}{a}+c$ **etc**

IV $\int\left[f(x)\right]^n.f'(x)\,dx=\frac{\left[f(x)\right]^{n+1}}{n+1}+c$

V $\int\left(\frac{f'(x)}{f(x)}\right)dx=\log|f(x)|+c$

VI $\int\left(\frac{f'(x)}{\sqrt{f(x)}}\right)dx=2\sqrt{f(x)}+c$

VII $\int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+c$

VIII $\int e^{f(x)}f'(x)dx=e^{f(x)}+c$

Key points

Unit I : Ordinary Differential Equations Of First Order And Of First Degree

Definition: An equation which involves differentials is called a Differential equation.

Ordinary differential equation: An equation is said to be ordinary if the derivatives have reference to only one independent variable.

(1) **Partial Differential equation:** A Differential equation is said to be partial if the derivatives in the equation have reference to two or more independent variables.

Order of a D.E equation: A Differential equation is said to be of order 'n' if the n^{th} derivative is the highest derivative in that equation.

Degree of a Differential equation: Degree of a D .Equation is the degree of the highest derivative in the equation after the equation is made free from radicals and fractions in its derivations.

Differential Equations of first order and first degree:

The general form of first order ,first degree DEquation is $\frac{dy}{dx} =$ f(x,y) or [Mdx + Ndy =0 Where M and N are functions of x and y]. There is no general method to solve any first order D.Equation. The equation which belong to one of the following types can be easily solved.

In general the first order D.Equation can be classified as:

1). Variable separable type

2). Homogeneous differential equation

3). Exact differential equations and

(a)equations reducible to exact equations.

(i) Integratin factor 1 (ii) Integrating Factor 2 (iii) Integrating Factor 3 (iv) Integrating Factor 4

4) Linear differential equation

(a) Bernoulli's linear differential equation.

(b) Equations reducible to linear form

Applications of first order and first degree differential equations:

1. Newton's law of cooling
2. Natural growth and Decay

I VARIABLE SEPARABLE:

If the D.equation $\frac{dy}{dx}$ =f(x,y) can be expressed of the form $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or f(x) dx –g(y)dy =0 where f and g are continuous functions of a single variable, then it is said to be of the form variable separable.

General soln of variable saparable is $\int f(x)dx - \int g(y)dy$= c

II HOMOGENEOUS DIFFERENTIAL EQUATION:

The diff. eq. $\frac{dy}{dx} = f(x,y)$ where $f(kx,ky) = f(x,y)$ is said to be homogeneous.

Working rule: $\frac{dy}{dx} = f(x,y)$-----(1)

purt y=vx, diff. wrt x gives $\frac{dy}{dx} = v + x\frac{dv}{dx}$

substituting above in (1), $v + x\frac{dv}{dx} = f(x,vx)$

on simplification of RHS, $v + x\frac{dv}{dx} = g(v)$

$$\Rightarrow \quad x\frac{dv}{dx} = g(v) - v$$

$$\Rightarrow \quad \frac{dv}{g(v) - v} = \frac{dx}{x}$$

Integrating on both sides we get the required solution.

Exact Differential Equations:

Def: Let M(x,y)dx +N(x,y) dy =0 be a first order and first degree differential equation where M & N are real valued functions of x,y . Then the equation Mdx + Ndy =0 is said to be an exact differential equation if ∃ a function *f* such that.

$$M = \frac{\partial f}{\partial x} \text{ and } N = \frac{\partial f}{\partial y}.$$

Necessary and sufficient condition for Exactness: If M(x,y) & N (x,y) are two real functions which have continuous partial derivatives then the necessary and sufficient condition for the Differential equation Mdx+ Ndy =0 is to be exact is that $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

Working Rule : Consider M(x,y)dx +N(x,y) dy =0.

step 1 check $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

step 2 find U= $\int^{x} Mdx$ (i.e. integrate M partially wrt *x* i.e. considering *y* as constant)

step 3 find V= $\int$[terms free from x in N]dy

step 4 Final solution is U+V=c, where c is a constant.

REDUCTION OF NON-EXACT DIFFERENTIAL EQUATIONS TO EXACT USING INTEGRATING FACTORS

Definition: If the Non-exact differential equation M(x,y) dx + N (x,y) dy = 0 can be made exact by multiplying it with a suitable function f (x,y) $\neq$ 0. Then this function is called an Integrating factor(I.F).

Some methods to find an I.F to a non-exact Differential Equation Mdx+N dy =0

Method-1: If M(x,y) dx + N (x,y) dy =0 is a homogeneous differential equation and Mx +Ny $\neq$ 0, then $\frac{1}{Mx+Ny}$ is an integrating factor of Mdx+ Ndy =0.

Method- 3: If the equation Mdx + N dy =0 is of the form y.f(xy) dx + xg(xy) dy = 0 (ie. M= y f(xy) and N=xg(xy)) & Mx- Ny $\neq$ 0 then $\frac{1}{Mx-Ny}$ is an integrating factor of Mdx+ Ndy =0.

Method -3: If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(x)\ or\ K$ then I.F. is $e^{\int f(x)dx} or\ e^{\int kdx}$

Method -4: If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)=g(y)\ or\ K$ then I.F. is $e^{\int g(y)dy} or\ e^{\int kdy}$.

LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER:

Def: An equation of the form $\frac{dy}{dx} + P(x).y = Q(x)$ is called a linear differential equation of first order in y.

Working Rule: To solve the liner equation $\frac{dy}{dx} + P(x).y = Q(x)$

step 1 Find find $\int p(x)dx$

step 2 Find the Integrating factor I.F = $e^{\int p(x)dx}$

step 3 Find $\int Q(x)e^{\int p(x)dx}dx$

step 4 General solution is $ye^{\int p(x)dx} = \int Q(x)e^{\int p(x)dx}dx + c$

Note: An equation of the form $\frac{dx}{dy} + P(y)x = Q(y)$ is called a linear differential equation of first order in x whose solution is $xe^{\int p(y)dy} = \int Q(y)e^{\int p(y)dy}dy + c$

BERNOULLI'S EQUATION :

(EQUATION'S REDUCIBLE TO LINEAR EQUATION)

Def: An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called Bernoulli's linear differential equation, where p & Q are function of x and n is a real constant.

Working Rule: $\frac{dy}{dx} + P(x)y = Q(x)y^n$ ……………………(1)

multiply the given equation (1) by y^{-n}

$$\Rightarrow \quad y^{-n}.\frac{dy}{dx} + P(x) . y^{1-n} = Q \text{ ……(2)}$$

Then take y^{1-n} = u

$$(1-n)\, y^{-n}.\frac{dy}{dx} = \frac{du}{dx}$$

$$\Rightarrow \quad y^{-n}.\frac{dy}{dx} = \frac{1}{1-n}\frac{du}{dx}$$

Then equation (2) becomes

$$\frac{1}{1-n}\frac{du}{dx} + P(x) . u = Q(x)$$

$\frac{du}{dx}$ + (1-n) P(x) u = (1-n)Q(x) which is linear in 'u' and hence we can solve it.

EQUATIONS REDUCIBLE TO LINEAR EQUATION

Consider differential equation of the form $f'(y)\frac{dy}{dx}+P(x)f(y)=Q(x)$ …..(1)

put f(y) = u and f'(y) $\frac{dy}{dx}$ = $\frac{du}{dx}$ in (1)

$\frac{du}{dx}$ +P(x)u = Q(x) which is linear in 'u' hence we can solve it.

APPLICATION OF DIFFERENTIAL EQUATIONS OF FIRST ORDER

NEWTON'S LAW OF COOLING

STATEMENT: The rate of change of the temp of a body is proportional to the difference of the temp of the body and that of the surrounding medium.

Let θ be the temp of the body at time 't' and θ_0 be the temp of its surrounding medium(usually air). By the Newton's low of cooling , we have

$$\frac{d\theta}{dt}\alpha(\theta-\theta_0) \text{ i.e. } \frac{d\theta}{dt}=-k(\theta-\theta_0) \text{ where } k \text{ is constant}$$

LAW OF NATURAL GROWTH OR DECAY

STATEMENT: Let x(t) or x be the amount of a substance at time ' t' and let the substance be getting converted chemically . A law of chemical conversion states that the rate of change of amount x(t) of a chemically changed substance is proportional to the amount of the substance available at that time

$$\frac{dx}{dt}\ \alpha\ x$$

$$\frac{dx}{dt}=-kx \text{ (in case of 'decay')}$$

$$\frac{dx}{dt}=kx \quad \text{(in case of 'growth')}$$

Where k is a constant of proportionality.

Unit II : HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

Def: An equation of the form $\frac{d^ny}{dx^n}$ + P_1(x) $\frac{d^{n-1}y}{dx^{n-1}}$ + P_2(x) $\frac{d^{n-2}y}{dx^{n-2}}$ + --------+ P_n(x) .y = Q(x)

Where P_1(x), P_2(x), P_3(x)… …..P_n(x) , Q(x) (functions of x) continuous is called a linear differential equation of order *n*.

LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS:

Def: An equation of the form $\frac{d^n y}{dx^n}$ + P_1 $\frac{d^{n-1}y}{dx^{n-1}}$ + P_2 $\frac{d^{n-2}y}{dx^{n-2}}$ + --------+ P_{n-1} . $\frac{dy}{dx}$ + P_n .y = Q(x) where $P_1, P_2, P_3, \ldots P_n$, are real constants and Q(x) is a continuous functions of x is called an Linear differential equation of order ‘ n’ with constant coefficients.

Note:

1. Operator D $= \frac{d}{dx}$; $D^2 = \frac{d^2}{dx^2}$; …………………… $D^n = \frac{d^n}{dx^n}$
2. Operator $\frac{1}{D}Q(x) = \int Q(x)dx$

<u>To find the general solution of f(D).y = 0</u> :

Here f(D) = $D^n + P_1 D^{n-1} + P_2 D^{n-2}$ +-----------+P_n is a polynomial in D.

Consider the auxiliary equation (A.E.), f(m) = 0

i.e $m^n + P_1 m^{n-1} + P_2 m^{n-2}$ +-----------+P_n = 0

Let the roots be $m_1, m_2, m_3, \ldots m_n$.

Depending on the nature of the roots we write the solution as follows:

Consider the following table

E.no	**Roots of A.E f(m) =0**	**Solution**
1.	$m_1, m_2, ..m_n$ are real and distinct.	$y = c_1e^{m_1x} + c_2e^{m_2x} + \ldots + c_ne^{m_nx}$
2.	m_1, m_2 are equal and real($m_1 = m_2 = m$ i.e repeated twice) &the rest are real and different.	$y = (c_1+c_2x)e^{mx} + c_3e^{m_3x} + \ldots + c_ne^{m_1x}$
	m_1, m_2, m_3 are equal and real($m_1, m_2, m3 = m$ i.e repeated thrice) &the rest are real and different.	$y = (c_1+c_2x+c_3x^2)e^{mx} + c_4e^{m_4x} + \ldots + c_ne^{m_nx}$
3.	Two roots are complex conjugate say $a \pm ib$ and rest are real and distinct.	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx) + c_3e^{m_3x} + \ldots + c_ne^{mnx}$
4.	If $a \pm ib$ are repeated twice & rest are real and distinct	$y = e^{ax}[(c_1+c_2x)\cos bx + (c_3+c_4x)\sin bx] + c_5e^{m_5x} + \ldots\ldots + c_ne^{m_nx}$

<u>General solution of f(D) y = Q(x)</u>

Its solutions is given by $y = y_c + y_p$

Where y_c is called Complementary Function (C.F.), which is the general solutions of f(D)y=0

Where y_p is called Particular Integral (P.I.) defined by

$y_p = \frac{1}{f(D)}Q(x)$

P.I. (1) of f (D) y = Q(x) where Q(x) =e^{ax} for (a) ≠ 0

Case1: when f(a) ≠ 0, P.I = $\frac{1}{f(D)}Q(x) = \frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$.

Case 2: whenIf f(a) = 0 Then f(D) = $(D-a)^k\,\phi(D)$

(i.e ' a' is a repeated root *k* times).

Then P.I $= \frac{1}{f(D)}e^{ax} = \frac{1}{\phi(D)(D-a)^k}e^{ax} = \frac{1}{\phi(a)}\frac{x^k}{k!}e^{ax}$ provided $\phi(a) \neq 0$

where $\frac{1}{(D-a)^k}e^{ax} = \frac{x^k}{k!}e^{ax}$

P.I. (2) of f(D) y =Q(x) where Q(x) = sin bx or cos bx where ' b ' is constant.

Case 1: $y_p = \frac{1}{f(D)}\sin bx$

In *f*(D) put $D^2 = -b^2$ if $\phi(-b^2) \neq 0$ where $f(D) = \phi(D^2)$.

$\rightarrow y_p = \frac{1}{\alpha D + \beta}\sin bx$ where α,β are constants

→ rationalize the denominator and put $D^2 = -b^2$ in the denominator

→ simplify the numerator using $D = \frac{d}{dx}$.

Case 2: If if $\phi(-b^2)=0$ then use the following two formulae:

$$\frac{1}{D^2+b^2}\sin bx = -\frac{x}{2b}\cos bx$$

$$\frac{1}{D^2+b^2}\cos bx = \frac{x}{2b}\sin bx$$

P.I. (3) for f(D) y = Q(x) where Q(x) = x^k where k is a positive integer

$y_p = \frac{1}{f(D)}x^k$

→ express *f*(D) =[1± ϕ(D)]

$\rightarrow \frac{1}{f(D)}x^k = \frac{1}{[1\pm\phi(D)]}x^k = [1\pm\phi(D)]^{-1}x^k$

$\rightarrow$ Then use the following formulae and D = $\frac{d}{dx}$ to simplify the above

$(1 - D)^{-1} = 1 + D + D^2 + D^3 +$ ------------------

$(1 + D)^{-1} = 1 - D + D^2 - D^3 +$ ------------------

P.I. (4) of f(D) y = Q(x) when Q(x) = e^{ax} V where 'a' is a constant and V is function of x. where V =sin bx or cos bx or x^k

Then P.I = $\frac{1}{f(D)}Q(x) \quad = \frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)}V$

$\rightarrow \frac{1}{f(D+a)}V$ can be evaluated depending on V using above P.I.s (2) to (3).

P.I. (5) of f(D) y = Q(x) when Q(x) = x V where V = sinbx or cosbx.

Then P.I = $\frac{1}{f(D)}Q(x) = \frac{1}{f(D)}xV \quad = \left[x - \frac{1}{f(D)}f'(D)\right]\frac{1}{f(D)}V$.

General Method of finding P.I.

P.I, $y_p = \frac{1}{f(D)}Q(x)$

$\rightarrow$Let f(D) = (D-α_1) (D-α_2) (D-α_3)……. (D-α_n)

$\rightarrow$ $y_p = \frac{1}{f(D)}Q(x) = \frac{1}{(D-a_1)\,(D-a_2)\,(D-a_3)\ldots\ldots(D-a_n)}Q(x)$

$= \frac{A_1}{(D-\alpha_1)} + \frac{A_2}{(D-\alpha_2)} + \ldots\ldots\ldots\ldots + \frac{A_n}{(D-\alpha_n)}$ and simplify each term using the following $\frac{1}{D-\alpha}Q(x) = e^{\alpha x}\int e^{-\alpha x}Q(x)dx$.

Apply the *method of variation of parameters* to solve $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

$\rightarrow$Consider $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, let its solution i.e. Complementary Function be

$y_c = C_1 u(x) + C_2 v(x)$

$\rightarrow$Let the Particular Integral be y_p =A(x) u(x) + B(x) v(x), where A(x) and B(x) are to be found.

$\rightarrow$ Find $u\frac{dv}{dx} - v\frac{du}{dx}$

$\rightarrow$A(x) and B(x) are given by

$$A(x) = -\int \left(\frac{vR}{u\frac{dv}{dx} - v\frac{du}{dx}} \right) dx \text{ and } B(x) = \int \left(\frac{uR}{u\frac{dv}{dx} - v\frac{du}{dx}} \right) dx$$

→ The general solution is y= $y_c + y_p = C_1u(x) + C_2\,v(x) + A(x)\,u(x) + B(x)\,v(x)$.

Unit III : Laplace Transforms

Laplace Transform:

Let *F*(t) be a function defined for all positive values of *t*, then the Laplace transform of F(t) denoted by L{*F*(t)} or $f(s)$ is defined by

$$L\{F(t)\} = f(s) = \int_0^\infty e^{-st} F(t)dt \quad \text{---------------(1)}$$

Here, F(t) is said to be inverse laplace transform of $f(s)$, which is written as $F(t) = L^{-1}\{f(s)\}$.

The symbol 'L' is called the laplace transform operator. The function *F*(t) must satisfy the following conditions for the existence of the laplace transform.

(a) The function *F*(t) must be piece-wise continuous in any limited interval 0<a≤t≤b.
(b) The function *F*(t) is of exponential order.

Standard Formulae

$L\{1\} = \frac{1}{s}$

$L\{k\} = \frac{k}{s}$

$L\{t\} = \frac{1}{s^2}$

$L\{t^n\} = \frac{n!}{s^{n+1}}$

$L\{e^{at}\} = \frac{1}{s-a}$

$L\{e^{-at}\} = \frac{1}{s+a}$

$L\{\cos at\} = \frac{s}{s^2+a^2}$ if s>0

$L\{\sin at\} = \frac{a}{s^2+a^2}$ if s>0

$L\{\cosh at\} = \frac{s}{s^2-a^2}$

$L\{\sinh at\} = \frac{a}{s^2-a^2}$.

1. First shifting theorem:

If $L\{F(t)\} = f(s)$ then $L\{e^{at}F(t)\} = f(s-a)$

Note: Unit Step Fucntion (OR) Heaviside's unit Function:

The unit step function is defined by H(t-a) or U(t-a) = $\begin{cases} 0, \text{ if } t<a \\ 1, \text{ if } t>a \end{cases}$

2. Second shifting theorem:

If $L\{F(t)\} = f(s)$ then $L\{F(t-a)H(t-a)\} = e^{-as} f(s)$.

(or)

If $L\{F(t)\} = f(s)$ and g(t) = $\begin{cases} F(t-a) & if\ t>a \\ 0 & if\ t<a \end{cases}$

then $L\{g(t)\} = e^{-as} f(s)$.

3. Change of scale property:

If $L\{F(t)\} = f(s)$ then $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$

4. Laplace transform of Derivatives:

If $L\{F(t)\} = f(s)$ then

5. Laplace transorm of Integrals:

If $L\{F(t)\} = f(s)$ then $L\left\{\int_0^t F(u)du\right\} = \frac{1}{s} f(s)$

similarly $L\left\{\int_0^t\int_0^t F(u)dudu\right\} = \frac{1}{s^2} f(s)$ and so on

6. Laplace transform of 'Multiples of *t*':

If $L\{F(t)\} = f(s)$ then $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$

7. Laplace transform of 'Division by *t*':

If $L\{F(t)\} = f(s)$ then $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)ds$

8. Laplace transform of Periodic function:

Note: A function F(t) is said to be periodic of period T if $F(t) = F(t+T) = F(t+2T) = ...$

Ex: *sin*t and *cos*t are periodic functions of 2π.

If *F*(t) is a periodic function of period T then $L\{F(t)\} = \frac{1}{1-e^{-sT}} \int_s^T e^{-st} F(t)dt$.

INVERSE LAPLACE TRANSFORM

If *f*(s) is the Laplace transform of a function *F*(t) then *F*(t) is called the inverse laplace transform of *f*(s) and it is denoted by $F(t) = L^{-1}\{f(s)\}$
where L^{-1} is called the inverse Laplace transform operator.

Table of Inverse Laplace transform:

S.no	*f*(s)	L^{-1}\{f(s)\} = *F*(t)
1	$\frac{1}{s}$	1
2	k	Ks
3	$\frac{1}{s^{n+1}}$	$\frac{t^n}{n!}$
4	$\frac{1}{s-a}$	e^{at}
5	$\frac{1}{s+a}$	e^{-at}
6	$\frac{1}{s^2+a^2}$	$\frac{1}{a}$ sinat
7	$\frac{s}{s^2+a^2}$	cosat
8	$\frac{1}{s^2-a^2}$	$\frac{1}{a}$ sinhat

9	$\frac{s}{s^2-a^2}$	coshat

Note: In finding Inverse Laplace transorm, it is required to use **Partial Fractions.**

Tips for partial fractions:

1. $\frac{1}{(s-a)(s-b)}=\frac{1}{(a-b)}\left(\frac{1}{(s-a)}-\frac{1}{(s-b)}\right)\frac{1}{(s^2-a)(s^2-b)}=\frac{1}{(a-b)}\left(\frac{1}{(s^2-a)}-\frac{1}{(s^2-b)}\right)$

2. $\frac{1}{(s^2-a)(s^2-b)}=\frac{1}{(a-b)}\left(\frac{1}{(s^2-a)}-\frac{1}{(s^2-b)}\right)$

3. $\frac{1}{(s^n-a)(s^n-b)}=\frac{1}{(a-b)}\left(\frac{1}{(s^n-a)}-\frac{1}{(s^n-b)}\right)$

In general use the following initial steps to resolve into partial fractions:

1. $\frac{1}{(s+a)(s+b)}=\frac{A}{s+a}+\frac{B}{s+b}$
2. $\frac{1}{s(s+a)(s+b)}=\frac{A}{s}+\frac{B}{s+a}+\frac{C}{s+b}$
3. $\frac{1}{(s+a)(s^2+b)}=\frac{A}{s+a}+\frac{Bc+D}{s^2+b}$
4. $\frac{1}{(s^2+a)(s^2+b)}=\frac{As+B}{(s^2+a)}+\frac{Cs+D}{(s^2+b)}$

5. $\frac{1}{(s+a)(s+b)^2} = \frac{A}{(s+a)} + \frac{B}{(s+b)} + \frac{C}{(s+b)^2}$

6. $\frac{1}{(s^2+a)(s+b)^2} = \frac{As+B}{(s^2+a)} + \frac{C}{(s+b)} + \frac{D}{(s+b)^2}$

1. First shifting theorem (Inverse):

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{f(s-a)\} = e^{at}F(t)$.

2. Second shifting theorem(Inverse):

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{e^{-as}f(s)\} = F(t-a)H(t-a)$.

(or)

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{e^{-as}f(s)\} = G(t)$

$$\textit{where } \mathrm{G(t)} = \begin{cases} F(t-a) & if\ \ t > a \\ 0 & if\ \ t < a \end{cases}$$

3. Change of scale property(Inverse):

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{f(as)\} = \frac{1}{a}F\left(\frac{t}{a}\right)$.

4. Inverse LT of Derivatives:

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{f^{(n)}(s)\} = (-1)^n t^n F(t)$, where $f^{(n)}(s) = \frac{d^n}{ds^n}[f(s)]$.

5. Inverse LT of Integrals:

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\left\{\int_0^\infty f(s)ds\right\} = \frac{F(t)}{t}$.

6. Inverse L T of 'Multiples of s':

If $L^{-1}\{f(s)\} = F(t)$ and $f^{(n)}(0) = 0$ for n=0,1,2,…n-1 then $L^{-1}\{s^n f(s)\} = F^{(n)}(t)$.

7. Inverse L T of 'Division by s':

If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\left\{\frac{f(s)}{s}\right\} = \int_0^t F(u)du$

Def: Convolution: If F(t), G(t) are two functions then convolution of the two functions is defined by

F(t) * G(t) = $\int_0^t F(u)G(t-u)du$.

8. Convolution Theorem:

If L{F(t)} = *f*(s) and L{G(t)} =*g* (s) then

L{F(t) * G(t)} = *f*(s).*g* (s) (**or**)

$L^{-1}\{f(s).g(s)\} = F(t) * G(t)$.

Application of Laplace Transform in solving Differential Equation:

Consider a linear differential equation

$$\frac{d^nY}{dt^n} + P_1 \frac{d^{n-1}Y}{dt^{n-1}} + P_2 \frac{d^{n-2}Y}{dt^{n-2}} + \text{--------} + P_{n-1} . \frac{dY}{dt} + P_n .Y = Q(t)$$

i.e. $Y^{(n)} + P_1 Y^{(n-1)} + P_2 Y^{(n-2)} + \text{--------} + P_{n-1} .Y' + P_n .Y = Q(t)$ ---------(1)

where $P_1, P_2, P_3, \ldots P_n$, are real constants and Q(t) is a continuous function of *t* with initial conditions $Y(0)=c_0, Y'(0)=c_1, \ldots\ldots Y^{n-1}(0)=c_{n-1}$.

working rule:

(1) Take laplace transform on both sides of (1)

(2) use the formulae

$L\{Y'(t)\} = sy(s) - Y(0)$

$L\{Y''(t)\} = s^2y(s) - sY(0) - Y'(0)$

:   :

:   :

$L\{Y^n(t)\} = s^n y(s) - s^{n-1}Y(0) - s^{n-2}Y'(0) \ldots\ldots - Y^{n-1}(0)$

(3) put $Y(0)=c_0, Y'(0)=c_1, \ldots\ldots Y^{n-1}(0)=c_{n-1}$

(4) Shift all terms with negative sign to right keeping $y(s)$ term alone left hand side.

(5) divide total equation by the coefficient of $y(s)$, keeping $y(s)$ alone left hand side and having a function of s on right hand side.

(6) Resolve the this function of s into partial fractions.

(7) Take Inverse Laplace Transform on both sides, that gives Y as a function of t, which is the required solution.

Unit IV : Vector Differentiation

Scalar point function:-

If to each point p(x,y,z) of a region in space there corresponds a definite scalar f(x,y,z) then 'f' is called a scalar point function and the region in which the scalar quantity is specified is called a scalar field.

⇒ Eg:- 1) Density of a body

2) Pressure of air in the earth's atmosphere.

Vector Point Function:-

If to each point p(x,y,z) of a region in space there corresponds a definite. Vector f(x,y,z) = f(p), then f is called a vector point function & the region in which 'f' is specified, is called a Vector Field.

Eg:- In distribution of velocity at all points of a moving fluid, velocity represents vector point function.

VECTOR DIFFERENTIAL OPERATOR

Def. The vector differential operator ∇(read as del) is defined as

$\nabla \equiv \bar{i}\frac{\partial}{\partial x}+\bar{j}\frac{\partial}{\partial y}+\bar{k}\frac{\partial}{\partial z}$. This operator possesses properties analogous to those of ordinary vectors as well as differentiation operator.

GRADIENT OF A SCALAR POINT FUNCTION

Let $\phi(x,y,z)$ be a scalar point function of position defined in some region of space. Then the vector function $\bar{i}\frac{\partial \phi}{\partial x}+\bar{j}\frac{\partial \phi}{\partial y}+\bar{k}\frac{\partial \phi}{\partial z}$ is known as the gradient of ϕ or $\nabla\phi$

$$\nabla\phi = \left(\bar{i}\frac{\partial}{\partial x}+\bar{j}\frac{\partial}{\partial y}+\bar{k}\frac{\partial}{\partial z}\right)\phi = \bar{i}\frac{\partial \phi}{\partial x}+\bar{j}\frac{\partial \phi}{\partial y}+\bar{k}\frac{\partial \phi}{\partial z}$$

Directional Derivative: The directional derivative of a scalar point function ϕ at a point P(x,y,z) in the direction of a unit vector e is equal to e. grad ϕ=e. $\nabla\phi$.

Level Surface:-

If $\phi(x,y,z)$ is a scalar point function which define a scalar field in a region R, the set of points (x,y,z) in space where $\phi(x,y,z)$ = constant is called a level surface of ϕ.

Eg:- $x^2 + y^2 + z^2 = c^2$, c>0 is level surfaces of the scalar field $\phi(x,y,z) = \sqrt{x^2 + y^2 + z^2}$

⇒ Geometrically, if f(x,y,z) = c represents a level surface of scalar field defined by f(x,y,z), then ∇f or grad f represents a surface outward at the point P and has the magnitude equal to the rate of change of f along this normal.

DIVERGENCE OF A VECTOR

Let $\bar{f}$ be any continuously differentiable vector point function. Then $\bar{i}.\frac{\partial \bar{f}}{\partial x}+\bar{j}.\frac{\partial \bar{f}}{\partial y}+\bar{k}.\frac{\partial \bar{f}}{\partial z}$ is called the divergence of $\bar{f}$ and is written as div $\bar{f}$.

i.e div $\bar{f} = \bar{i}.\frac{\partial \bar{f}}{\partial x}+\bar{j}.\frac{\partial \bar{f}}{\partial y}+\bar{k}.\frac{\partial \bar{f}}{\partial z} = \left(\bar{i}\frac{\partial}{\partial x}+\bar{j}\frac{\partial}{\partial y}+\bar{k}\frac{\partial}{\partial z}\right).\bar{f}$

hence we can write div $\bar{f}$ as div $\bar{f} = \nabla.\bar{f}$. This is a scalar point function.

- A vector point function $\bar{f}$ is said to be $\bar{f}$ solenoidal if div $\bar{f}$ =0.

CURL OF A VECTOR: Let $\bar{f}$ be any continuously differentiable vector point function. Then the vector function defined by $\bar{i}x\frac{\partial\bar{f}}{\partial x}+\bar{j}x\frac{\partial\bar{f}}{\partial y}+\bar{k}x\frac{\partial\bar{f}}{\partial z}$ is called curl of $\bar{f}$ and is denoted by curl $\bar{f}$ or (∇x $\bar{f}$).

$$\text{Curl } \bar{f} = \bar{i}x\frac{\partial\bar{f}}{\partial x}+\bar{j}x\frac{\partial\bar{f}}{\partial y}+\bar{k}x\frac{\partial\bar{f}}{\partial z}=\sum\left(\bar{i}x\frac{\partial\bar{f}}{\partial x}\right)$$

- $$curl\,\bar{f} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$
- A vector $\bar{f}$ is said to be Irrotational if curl $\bar{f} = \bar{0}$.
- If $\bar{f}$ is Irrotational, there will always exist a scalar function φ(x,y,z) such that $\bar{f}$ =grad ϕ. This is called scalar potential of $\bar{f}$.

Unit V :Vector Integration

1. Line integral:- (i) $\int_c \bar{F}.d\bar{r}$ is called Line integral of $\bar{F}$ along c

Note : Work done by $\bar{F}$ along a curve c is $\int_c \bar{F}.d\bar{r}$

2. Surface integral: $\int_c \bar{F}.\bar{n}ds$ is called surface integral.

3. Volume integral : Let V be the volume bounded by a surface $\bar{r} = \bar{f}$ (u,v). Let $\bar{F}$ ($\bar{r}$) be a vector point function define over V. Then the volume integral of $\bar{F}$ ($\bar{r}$) in the region V is denoted by $\int_V \bar{F}(\bar{r})\,dv$ or $\int_V \bar{F}dv$.

I. GAUSS'S DIVERGENCE THEOREM

(Transformation between surface integral and volume integral)

Let S be a closed surface enclosing a volume v. if $\bar{F}$ is a continuously differentiable vector point function, then

$$\int_V div\,\bar{F}dv = \int_s \bar{F}.\bar{n}\ dS$$

When $\bar{n}$ is the outward drawn normal vector at any point of S.

II. GREEN'S THEOREM IN A PLANE

(Transformation Between Line Integral and doouble Integral)

If S is Closed region in xy plane bounded by a simple closed curve C and if M and N are continuous functions of x and y having continuous derivatives in R, then

$$,\oint_C Mdx + Ndy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$$

Where C is traversed in the positive(anti clock-wise) direction.

III. STOKE'S THEOREM

(Transformation between Line Integral and Surface Integral)

Let S be a open surface bounded by a closed, non intersecting curve G. if $\bar{F}$ is any differentiable vector point function then $\oint_C \bar{F}.d\bar{r} = \int_S curl\bar{F}.\bar{n}ds$ where C is traversed in the positive direction and $\bar{n}$ is unit outward normal vector at any point of the surface.

UNIT WISE QUESTION BANK

UNIT – I: First Order ODE

I. Short Answer questions

1 .Solve $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

2. Solve $xy(1+xy^2)\frac{dy}{dx} = 1$

3. Find the orthogonal trajectory of the family of curves $ay^2 = x^3$, where a is variable

parameter.

4. The number x of bacteria in a culture grow at a rate proportional to x. The value of x was initially 50 and increased to 150 in one hour what will be the value of x after $1\frac{1}{2}$ hour.
5. Write the statements of Newton's Law of cooling, Natural growth and Decay.

II. Long answer Questions

1. Solve $x\frac{dy}{dx} + y = \log x$
2. Solve $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$
3. Solve $(1 + 2xy\cos x^2 - 2xy)\,dx + (\sin x^2 - x^2)\,dy = 0$
4. Solve $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^4)\,dy = 0$
5. Solve $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$
6. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$
7. Solve $(y\log y)\,dx + (x - \log y)\,dy = 0$
8. Solve $2xy\,dy - (x^2 - y^2 + 1)\,dx = 0$
9. If the temperature of air is 20^0c and the temperature of the body drops from 100^0c to 80^0c in 10 minutes, what will be its temperature after 20 minutes? When will the temperature be 40^0c?

III. Each question carries ½ mark.

1. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the condition that y=1 when x=1 is [ ]

   a) $4xy = x^3+3$ b) $4xy = x^4+3$ c) $4xy = y^4+3$ d) None

2. The family of straight lines passing through the origin is represented by the differential equation [ ]

   a) ydx+xdy=0 b)xdy-ydx=0 c) xdx+ydx=0 d) ydy-xdx=0

3. The differential equation satisfying the relation x=Acos(mt-α) is [ ]

   a) $\frac{dx}{dt} = 1 - x^2$ b) $\frac{d^2x}{dt^2} = -\alpha^2 x$ c) $\frac{d^2x}{dt^2} = -m^2 x$ D) $\frac{dx}{dt} = -m^2 x$

4. The equation $\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$ is [ ]

   a) Homogeneous b) Variable separable c) Exact d) None

5. Find the differential equation of the family of cardioids r=a(1+cosθ) is [ ]

a) $\frac{dr}{d\theta}+r\sin x=0$ b) $\frac{dr}{d\theta}+r\tan\left(\frac{\theta}{2}\right)=0$ c) $\frac{dr}{d\theta}+r\sin\left(\frac{\theta}{2}\right)=0$ d) None

UNIT-II : Ordinary Differential Equations of Higher Order

I. Short Answer questions

1Solve $D^2(D^2+9)=sin2x+5$

2. Solve D(D+5)+6 =100

3. Find the value of $\frac{1}{D^2+4}\sin 2x$

4. Find C.F of $(D+1)(D-2)^2y = e^{3x}$.

II. Long answer Questions

1) Solve $(D^2+1)\ y=cosx$ by the method of variation of parameters.
2) Solve $\frac{d^2y}{dx^2}+y=e^{-x}+x^3+e^x sinx.$
3) Solve $(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$

III. Each question carries ½ mark.

1) $\frac{1}{f(D)}[x\ v(x)]=$ ____
2) The solution of the D.E. $(D^3+3D)y=0$ is ____
3) $\frac{e^{-x}}{(D+1)^2}=$

a) $\frac{xe^{-x}}{2}$ b) $\frac{e^{-x}}{4}$ c) $\frac{e^{-x}}{2}$ d) $\frac{x^2e^{-x}}{2}$ [ ]

4) $\frac{1}{D+1}(1+e^x)=$ [ ]

(a) Cosx (b) sinx (c) cosecx (d) secx

UNIT – III: Laplace Transforms

I.Short Answer questions

1. Define Laplace transform of a function.
2. Define inverse Laplace transform of a function.
3. Define convolution of two functions.
4. Prove that L[t]= $\frac{1}{s^2}$

5. Prove that L[t^n]= $\frac{n!}{s^{n+1}}$
6. Find L[e^{at}]
7. Find L[sin at]
8. Prove that the function f(t)= 7^2 is exponential order 3.
9. Find L[sin (wt+α)]
10. Find L[sin2t+cos3t]g to polar coordinates

II. Long answer Questions

1. Find $L^{-1}\left[\frac{s}{s^4+4a^4}\right]$
2. Find $L^{-1}\left[\frac{s+3}{s^2-10s+29}\right]$
3. Find $L^{-1}\left[\frac{s}{s^2-a^2}\right]$
4. Find $L^{-1}\left[\frac{1+e^{-\pi s}}{s^2+1}\right]$
5. Using Laplace transforms method, solve (D^2 +1)y=6cos2t,t>0
6. Using Laplace transforms, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$, given that y(0)=0,$y^1$(0)=1

III Each question carries ½ mark.

1. $L^{-1}[1]=$ [ ]

   a) δt b) 1 c) 0 d) δ(t-1)

2. $L[te^{-at}]=$ [ ]

   a) $\frac{-1}{(s-a)^2}$ b) $\frac{1}{(s-a)^2}$ c) $\frac{1}{(s+a)^2}$ d) $\frac{-1}{(s+a)^2}$

3. If $L[f(t)]=\frac{3s}{6s^2+24}$, $f(t)=$ [ ]

a) $\frac{1}{2}\cos 2t$ b) $\frac{1}{3}\cos 2t$ c) $\frac{1}{3}\sin 2t$ d) $\frac{1}{2}\sin 2t$

4. $L^{-1}\left[\frac{2s}{(s+1)^2}\right] =$ [ ]

a) tcost b) –tsint c) –tcost d) tsint

5. $L\left[\int_0^t \frac{\sin t}{t}\,dt\right] =$ [ ]

a) $\tan^{-1}(s)$ b) $\cot^{-1}s$ c) $\frac{\tan^{-1}s}{s}$ d) $\frac{\cot^{-1}s}{s}$

UNIT – IV: Vector Differentiation

I. Short Answer questions

1. If $\overline{f} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+\lambda z)\bar{k}$ is solenoidal, find the value of λ.
2. Define irrotational, Solenoidal
3. Write the formulas of Curl, Div, Gradient

II. Long answer Questions

1. Find the work done in moving a particle in the force field $\overline{F} = 3x^2\bar{i} + (2xz-y)\bar{j} + z\bar{k}$.
2. Show that the vector $\overline{f} = (x^2-yz)\bar{i} + (y^2-zx)\bar{j} + (z^2-xy)\bar{k}$ is irrotational and find its scalar potential.
3. If Φ *and* Ψ are scalar functions, then prove that $\nabla\Phi \times \nabla\Psi$ is solenoidal.

III. Each question carries ½ mark.

1. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $curl\ \bar{r} =$

   a) $\bar{0}$ b) 0 c)1 d)3

2. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla^2(\log r) =$

   a) $\bar{0}$ b) 0 c) $\frac{1}{r^2}$ d)x+y+z

3. Physical interpretation of $\nabla\Phi$ is that______________________.

UNIT – V: Vector Integration

I.Short Answer questions

1. If $\overline{F} = 3xy\bar{i} - y^2\bar{j}$ evaluate $\int_C \overline{F}\cdot\overline{dr}$ where C is the curve $y = 2x^2$ in the XY-plane from (0,0) to (1,2)

2. Using Green's theorem evaluate $\int_C (2xy - x^2)\, dx + (x^2 + y^2)\, dy,$ where C is the closed curve of the region bounded by $y = x^2$ *and* $y^2 = x.$

3. If $\overline{F} = y\bar{i} + (x - 2xz)\bar{j} - xy\bar{k},$ evaluate $\int_S (\nabla \times \overline{F}) \cdot \bar{n}\, ds$ using Stokes theorem, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, above the XY-plane.

II. Long answer Questions

1. Verify Gauss's divergence theorem for $2x^2y\,\bar{i} - y^2\,\bar{j} + 4xz^2\,\bar{k}$ taken over the region of the first octant of the cylinder $y^2 + z^2 = 9$ *and* $x = 2.$
2. Verify Green's theorem for $\int_C (xy + y^2)\,dx + x^2 dy$, where C is bounded by $y = x$ *and* $y = x^2.$
3. Verify Stokes theorem for $\overline{F} = (2x - y)\,\bar{i} - y\,z^2\,\bar{j} + y^2 z\bar{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by projection of the xy-plane.

III. Each question carries ½ mark.

1. For any closed surface S, $\iint_S curl\overline{F} \cdot \bar{n}\, ds =$

   a) $\oint \overline{F} \cdot \overline{dr}$ b) 0 c) 1 d) $\oint \overline{F} \times \overline{dr}$

2. If $\oint \overline{F} \cdot \overline{dr}$ is independent of the path joining any two points if and only if it is ____________________.
3. The value of $\int_S \bar{r} \cdot \bar{n}\, dS$ is____________________.